一种新的用正交列阵构造纠缠态的方法

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摘要 对于一个 N 粒子多体纯态, 如果它的所有 k 粒子约化态是最大混合的, 则称为 k 级均匀态. 最近Goyeneche和Zyczkowski提出了一个通过正交列阵构造 k 级均匀态的方法. 本文将提出一种用正交列阵构造量子态的新方法, 即增加相位参数的方法. 采用同一正交列阵, 由我们的新方法所构造的量子态, 对于四粒子量子态, 发现该量子态不仅是1级均匀态, 而且为最大纠缠态; 对于五粒子量子态, 发现可以使原来的1级均匀态变为2级均匀态. 特别地, 对于四粒子量子态, 用新方法可以构造出许多新形式的最大纠缠态.

关键词 正交列阵, k 级均匀态, 四粒子最大纠缠态

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1 引言

量子纠缠是量子信息中的一个热门话题, 在量子通信与量子计算中有着极其重要的应用, 如量子隐形传态(Quantum Teleportation)^[1-6]、量子密钥分配(Quantum Key Distribution)^[7-10]、量子安全直接通信(Quantum Secure Direct Communication)^[11-17]等. 2013年, Arnaud 和Cerf^[18]定义了多粒子纠缠态的 k 级均匀态(k -uniform States), 对于一个 N 粒子多体纯态, 如果它的所有 k 粒子约化态是最大混合的, 则称为 k 级均匀态. 例如, GHZ态属于1级均匀态, 而W态不属于任何均匀态, 它的建立是量子信息理论中的一个重大进展, 在量子密集编码(Dense Coding)和纠错码(Error-correcting Codes)方面有十分重

要的应用^[19]. 2014年, Goyeneche 和Zyczkowski^[19]由正交列阵出发, 构造了2级均匀态(2-uniform States); 最近Yu等人^[20]通过正交列阵方法, 构造出粒子数为11时的3级均匀态. 本文将采取新的方法来构造量子态, 首先, 通过求极小值构造出四粒子最大纠缠态; 其次, 对于五粒子量子态, 用类似方法将原来的1级均匀态变为2级均匀态; 特别地, 对于四粒子量子态, 我们可得到许多新形式的最大纠缠态.

2 四粒子纠缠态

对四粒子最大纠缠态, 文献^[21, 22]给出了其满足的条件, 即

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$$\pi_{av} = \frac{1}{3}(\pi_{12} + \pi_{13} + \pi_{14}) = \frac{1}{3}, \quad (1)$$

其中 $\pi_{12} = \text{Tr}_{12}\rho_{12}^2$, $\pi_{13} = \text{Tr}_{13}\rho_{13}^2$, $\pi_{14} = \text{Tr}_{14}\rho_{14}^2$, $\rho_{12}, \rho_{13}, \rho_{14}$ 为约化密度矩阵, 且满足

$$\rho_{12} = \text{Tr}_{34}|\phi_{1234}\rangle\langle\phi_{1234}|, \rho_{13} = \text{Tr}_{24}|\phi_{1234}\rangle\langle\phi_{1234}|,$$

$$\rho_{14} = \text{Tr}_{23}|\phi_{1234}\rangle\langle\phi_{1234}|.$$

到目前为止, 我们发现3种形式的四粒子最大纠缠态.

第一种形式为 $(\frac{1}{4}, \frac{1}{4}, \frac{1}{2})$, 即 $\pi_{1i} = \frac{1}{4}$, $\pi_{1j} = \frac{1}{4}$, $\pi_{1k} = \frac{1}{2}$ (其中 $i \neq j \neq k$ 且 $i \neq 1, j \neq 1, k \neq 1$), 如

$$|\psi\rangle_{1234} = \frac{1}{2\sqrt{2}}(|0000\rangle - |0011\rangle - |0101\rangle + |0110\rangle + |1001\rangle + |1010\rangle + |1100\rangle + |1111\rangle)_{1234}. \quad (2)$$

这个态由Yeo和Chua^[23]提出, 并成为四粒子最大纠缠态, 可以得到 $\pi_{12} = \frac{1}{4}$, $\pi_{13} = \frac{1}{4}$, $\pi_{14} = \frac{1}{2}$ ^[22].

四粒子cluster态为^[24]

$$|\psi\rangle_{1234} = \frac{1}{2}(|0000\rangle + |0101\rangle + |1010\rangle - |1111\rangle)_{1234}. \quad (3)$$

可以得到 $\pi_{12} = \frac{1}{4}$, $\pi_{13} = \frac{1}{2}$, $\pi_{14} = \frac{1}{4}$ ^[22].

第二种形式为 $(\frac{3}{8}, \frac{3}{8}, \frac{1}{4})$, 即 $\pi_{1i} = \frac{3}{8}$, $\pi_{1j} = \frac{3}{8}$, $\pi_{1k} = \frac{1}{4}$ (其中 $i \neq j \neq k$ 且 $i \neq 1, j \neq 1, k \neq 1$), 如

$$|\psi\rangle_{1234} = \frac{1}{2}(|0000\rangle + |011\rangle + |1101\rangle + |-110\rangle)_{1234}, \quad (4)$$

其中 $|+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$, $|-\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$, 这个态由Brown等人^[25]提出, 可以得到 $\pi_{12} = \frac{3}{8}$, $\pi_{13} = \frac{1}{4}$, $\pi_{14} = \frac{3}{8}$ ^[22].

第三种形式为 $(\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$, 即 $\pi_{1i} = \frac{1}{3}$, $\pi_{1j} = \frac{1}{3}$, $\pi_{1k} = \frac{1}{3}$ (其中 $i \neq j \neq k$ 且 $i \neq 1, j \neq 1, k \neq 1$), 如

$$|\psi\rangle_{1234} = \frac{1}{\sqrt{6}}(|0011\rangle + |1100\rangle + \omega(|0101\rangle + |1010\rangle) + \omega^2(|0110\rangle + |1001\rangle))_{1234}, \quad (5)$$

其中 $\omega = e^{i\frac{2\pi}{3}}$, 这个态由Higuchi和Sudbery^[26]提出, 可以得到 $\pi_{12} = \frac{1}{3}$, $\pi_{13} = \frac{1}{3}$, $\pi_{14} = \frac{1}{3}$ ^[22].

在构造新的量子态之前, 先回顾对于四粒子, 如何利用正交列阵构造1级均匀态, 文献^[19]给出的正交列阵如下:

$$\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 \end{pmatrix}.$$

由此构成的四粒子态如下:

$$|\phi_4\rangle = \frac{1}{2\sqrt{2}}(|0000\rangle + |0011\rangle + |0101\rangle + |0110\rangle + |1001\rangle + |1010\rangle + |1100\rangle + |1111\rangle)_{1234}. \quad (6)$$

若一个 $r \times N$ 的矩阵 S , 由集合 $M = \{m_1, m_2, \dots, m_d\}$ 中元素构成, 且满足 S 的任意 $r \times N$ 子列中每行含有所有的 k 元数组, 每个 k 元数组重复次数相同, 这样的矩阵 S , 称之为正交列阵, 表示为 $OS(r, N, d, k)$, 其中 r 和 N 分别表示正交列阵的行数和列数; d 表示不同数字出现的个数, 即维度; k 表示强度.

根据建立 k 级均匀态所需的基本条件^[20]:

(1) 正交列阵的任意 k 列组成的子列阵中每行含有由 M 集合中元素组成的所有 k 元数组, 且每个 k 元数组重复相同的次数;

(2) 正交列阵的任意 $N - k$ 列组成的子列阵中每行含有 $(N - k)$ 元数组, 且每个 $(N - k)$ 元数组互不重复.

对于满足以上两个重要条件的正交列阵, 可以找到该正交列阵所对应的 k 级均匀态.

根据上面的条件, 当去掉第一列(列 i)时, 情况如下: 剩余的三列, 也就是第二、三、四列(列 jk)所形成的子矩阵均由互不重复的三元数组(000, 011, 101, 110, 001, 010, 100, 111)构成, 且去掉的这一列(列 i)由一元数组(0和1)构成, 并各重复四遍; 同理当去掉第二列(列 j)时, 情况如下: 剩余的三列, 也就是第一、三、四列(列 ik)所形成的子矩阵均由互不

重复的三元数组(000, 011, 001, 010, 101, 110, 100, 111)构成, 且去掉的这一列(列*j*)由一元数组(0和1)构成, 并各重复4遍; 同样地, 当去掉第三或第四列也会得到类似的结果(如下子矩阵).

<i>i</i>	<i>j</i>	<i>k</i>	<i>l</i>	<i>j</i>	<i>i</i>	<i>k</i>	<i>l</i>
0	0	0	0	0	0	0	0
0	0	1	1	0	0	1	1
0	1	0	1	1	0	0	1
0	1	1	0	1	0	1	0
1	0	0	1	0	1	0	1
1	0	1	0	0	1	1	0
1	1	0	0	1	1	0	0
1	1	1	1	1	1	1	1

由上式可以看出该四粒子态是一个1级均匀态.

另外, 根据式(6)有 $\rho_{12} = \text{Tr}_{34}|\psi_4\rangle\langle\psi_4|$, $\pi_{12} = \text{Tr}_{12}\rho_{12}^2$, 可得 $\pi_{12} = \frac{1}{2}$, 同理可得 $\pi_{13} = \pi_{14} = \frac{1}{2}$.

对于上面的正交列阵, 我们采用新的方法所构造的四粒子量子态如下:

$$|\psi_4\rangle = \frac{1}{2\sqrt{2}}(|0000\rangle + e^{i\varphi_3}|0011\rangle + e^{i\varphi_5}|0101\rangle + e^{i\varphi_6}|0110\rangle + e^{i\varphi_9}|1001\rangle + e^{i\varphi_{10}}|1010\rangle + e^{i\varphi_{12}}|1100\rangle + e^{i\varphi_{15}}|1111\rangle)_{1234}. \quad (7)$$

由 $\rho_1 = \text{Tr}_{234}|\psi_4\rangle\langle\psi_4|$, $\pi_1 = \text{Tr}_1\rho_1^2$ 可得 $\pi_1 = \frac{1}{2}$, 同理可得 $\pi_2 = \pi_3 = \pi_4 = \frac{1}{2}$.

所以这个四粒子态也是一个1级均匀态.

进一步, 我们可得出

$$\begin{aligned} \pi_{12} &= \text{Tr}_{12}\rho_{12}^2 = \frac{1}{32}[12 + 2\cos(\varphi_3 + \varphi_{12} - \varphi_{15}) \\ &\quad + 2\cos(\varphi_5 + \varphi_{10} - \varphi_6 - \varphi_9)], \\ \pi_{13} &= \text{Tr}_{13}\rho_{13}^2 = \frac{1}{32}[12 + 2\cos(\varphi_5 + \varphi_{10} - \varphi_{15}) \\ &\quad + 2\cos(\varphi_3 + \varphi_{12} - \varphi_6 - \varphi_9)], \\ \pi_{14} &= \text{Tr}_{14}\rho_{14}^2 = \frac{1}{32}[12 + 2\cos(\varphi_6 + \varphi_9 - \varphi_{15}) \\ &\quad + 2\cos(\varphi_3 + \varphi_{12} - \varphi_5 - \varphi_{10})]. \end{aligned}$$

由四粒子最大纠缠态所满足的条件^[21, 22], 函数 $f(\varphi_3, \varphi_5, \varphi_6, \varphi_9, \varphi_{10}, \varphi_{12}, \varphi_{15})$ 取最小值, 通过计算,

可得

$$\begin{aligned} f(\varphi_3, \varphi_5, \varphi_6, \varphi_9, \varphi_{10}, \varphi_{12}, \varphi_{15}) \\ = \cos(\varphi_3 + \varphi_{12} - \varphi_{15}) + \cos(\varphi_6 + \varphi_9 - \varphi_{15}) \\ + \cos(\varphi_5 + \varphi_{10} - \varphi_{15}) + \cos(\varphi_3 + \varphi_{12} - \varphi_6 \\ - \varphi_9) + \cos(\varphi_3 + \varphi_{12} - \varphi_5 - \varphi_{10}) + \cos(\varphi_5 \\ + \varphi_{10} - \varphi_6 - \varphi_9) = -2. \end{aligned}$$

令 $\varphi_{15} = 0$, $\varphi_6 + \varphi_9 = \theta$, $\varphi_3 + \varphi_{12} = \pi$, $\varphi_5 + \varphi_{10} = \pi + \theta$, 可得 $f(\varphi_3, \varphi_5, \varphi_6, \varphi_9, \varphi_{10}, \varphi_{12}, \varphi_{15}) = -2$, 且有

$$\begin{aligned} \pi_{12} &= \text{Tr}_{12}\rho_{12}^2 = \frac{1}{4}, \\ \pi_{13} &= \text{Tr}_{13}\rho_{13}^2 = \frac{1}{32}[12 - 4\cos\theta], \\ \pi_{14} &= \text{Tr}_{14}\rho_{14}^2 = \frac{1}{32}[12 + 4\cos\theta]. \end{aligned}$$

若 $\theta = 0$, 则 $\pi_{12} = \frac{1}{4}$, $\pi_{13} = \frac{1}{4}$, $\pi_{14} = \frac{1}{2}$, 正是第一种形式最大纠缠态.

例如, 若 $\theta = 0$, 令 $\varphi_6 = 0$, $\varphi_9 = 0$, $\varphi_3 = \frac{\pi}{2}$, $\varphi_{12} = \frac{\pi}{2}$, $\varphi_5 = \frac{\pi}{2}$, $\varphi_{10} = \frac{\pi}{2}$, 则可得

$$|\psi_4\rangle = \frac{1}{2\sqrt{2}}(|0000\rangle + i|0011\rangle + i|0101\rangle + |0110\rangle + |1001\rangle + i|1010\rangle + i|1100\rangle + |1111\rangle)_{1234}.$$

若 $\theta = \frac{\pi}{2}$, 则 $\pi_{12} = \frac{1}{4}$, $\pi_{13} = \frac{3}{8}$, $\pi_{14} = \frac{3}{8}$, 正是第二种形式最大纠缠态.

例如, 若 $\theta = \frac{\pi}{2}$, 令 $\varphi_6 = \frac{\pi}{4}$, $\varphi_9 = \frac{\pi}{4}$, $\varphi_3 = \frac{3\pi}{4}$, $\varphi_{12} = \frac{\pi}{4}$, $\varphi_5 = \pi$, $\varphi_{10} = \frac{\pi}{2}$, 则可得

$$\begin{aligned} |\psi_4\rangle &= \frac{1}{2\sqrt{2}}\left(|0000\rangle + \left(-\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i\right)|0011\rangle \right. \\ &\quad - |0101\rangle + \left(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i\right)|0110\rangle \\ &\quad + \left(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i\right)|1001\rangle + i|1010\rangle \\ &\quad \left. + \left(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i\right)|1100\rangle + |1111\rangle\right)_{1234}. \end{aligned}$$

若 θ 为其他值时, 则为新形式的最大纠缠态. 例如, 当 $\theta = \frac{\pi}{3}$ 时, $\pi_{12} = \frac{1}{4}$, $\pi_{13} = \frac{5}{16}$, $\pi_{14} = \frac{7}{16}$.

例如, 若 $\theta = \frac{\pi}{3}$, 令 $\varphi_6 = \frac{\pi}{3}$, $\varphi_9 = 0$, $\varphi_3 = \frac{\pi}{2}$, $\varphi_{12} = \frac{\pi}{2}$, $\varphi_5 = \frac{\pi}{3}$, $\varphi_{10} = \pi$, 则可得

$$|\psi_4\rangle = \frac{1}{2\sqrt{2}} \left(|0000\rangle + i|0011\rangle + \left(\frac{1}{2} + \frac{\sqrt{3}}{2}i\right)|0101\rangle + \left(\frac{1}{2} + \frac{\sqrt{3}}{2}i\right)|0110\rangle + |1001\rangle - |1010\rangle + i|1100\rangle + |1111\rangle \right)_{1234}.$$

因此, 我们可得到许多新形式的四粒子最大纠缠态.

3 五粒子纠缠态

对于五粒子, 文献[19]给出的正交列阵如下:

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 & 0 \end{pmatrix}.$$

由此构成的五粒子态如下:

$$|\phi_5\rangle = \frac{1}{2\sqrt{2}} (|00000\rangle + |00101\rangle + |01010\rangle + |01111\rangle + |10011\rangle + |10110\rangle + |11001\rangle + |11100\rangle)_{12345}. \quad (8)$$

同理根据建立 k 级均匀态所需的基本条件 [20], 容易判断上述五粒子态也是一个 1 级均匀态.

另外, 根据式(8)我们有 $\rho_{12} = \text{Tr}_{345}|\phi_5\rangle\langle\phi_5|$, $\pi_{12} = \text{Tr}_{12}\rho_{12}^2$, 可得 $\pi_{12} = \frac{1}{4}$, 同理可得 $\pi_{13} = \pi_{14} = \pi_{15} = \pi_{23} = \pi_{25} = \pi_{34} = \pi_{45} = \frac{1}{4}$, 其中 $\pi_{24} = \pi_{35} = \frac{1}{2}$.

对于上面的正交列阵, 我们采用新的方法所构造的五粒子量子态如下:

$$|\psi_5\rangle = \frac{1}{2\sqrt{2}} (|00000\rangle + e^{i\varphi_5}|00101\rangle + e^{i\varphi_{10}}|01010\rangle + e^{i\varphi_{15}}|01111\rangle + e^{i\varphi_{19}}|10011\rangle + e^{i\varphi_{22}}|10110\rangle$$

$$+ e^{i\varphi_{25}}|11001\rangle + e^{i\varphi_{28}}|11100\rangle)_{12345}. \quad (9)$$

进一步, 我们可得出

$$\begin{aligned} \pi_{12} &= \pi_{13} = \pi_{14} = \pi_{15} = \pi_{23} = \pi_{25} = \pi_{34} = \pi_{45} = \frac{1}{4}, \\ \pi_{24} &= \text{Tr}_{24}\rho_{24}^2 = \frac{1}{32} [12 + 2\cos(\varphi_{15} - \varphi_5 - \varphi_{10}) \\ &\quad + 2\cos(\varphi_{22} - \varphi_{28} + \varphi_{25} - \varphi_{19})], \\ \pi_{35} &= \text{Tr}_{35}\rho_{35}^2 = \frac{1}{32} [12 + 2\cos(\varphi_{15} - \varphi_5 - \varphi_{10}) \\ &\quad + 2\cos(\varphi_{22} - \varphi_{28} + \varphi_{25} - \varphi_{19})]. \end{aligned}$$

若 $\cos(\varphi_{15} - \varphi_5 - \varphi_{10})$ 和 $\cos(\varphi_{22} - \varphi_{28} + \varphi_{25} - \varphi_{19})$ 取最小值, 即 $\cos(\varphi_{15} - \varphi_5 - \varphi_{10}) = -1$, $\cos(\varphi_{22} - \varphi_{28} + \varphi_{25} - \varphi_{19}) = -1$, 进一步可得 $\varphi_{15} - \varphi_5 - \varphi_{10} = \pi$, $\varphi_{22} - \varphi_{28} + \varphi_{25} - \varphi_{19} = \pi$, 则 $\pi_{24} = \pi_{35} = \frac{1}{4}$.

例如, 令 $\varphi_{15} = 2\pi$, $\varphi_5 = 0$, $\varphi_{10} = \pi$, $\varphi_{22} = \frac{3\pi}{2}$, $\varphi_{28} = \pi$, $\varphi_{25} = \frac{3\pi}{4}$, $\varphi_{19} = \frac{\pi}{4}$, 则可得

$$|\psi_5\rangle = \frac{1}{2\sqrt{2}} (|00000\rangle + |00101\rangle - |01010\rangle + |01111\rangle + \left(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i\right)|10011\rangle - i|10110\rangle + \left(-\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i\right)|11001\rangle - |11100\rangle)_{12345}.$$

则这个五粒子量子态是一个 2 级均匀态, 根据五粒子最大纠缠态满足的条件 [21], 即 $\pi_{av} = \frac{1}{4}$, 显然满足此条件有很多取值, 因此可得到许多新形式的五粒子最大纠缠态.

4 结论

本文提出了由正交列阵构造最大量子纠缠态的新方法, 对于四粒子量子态, 得到的量子态不仅是 1 级均匀态, 而且为最大纠缠态, 特别地, 我们采用新方法可以构造出许多新形式的四粒子最大纠缠态; 同样地对于五粒子量子态, 由相同的正交列阵, 将文献[19]得出的 1 级均匀态变为 2 级均匀态, 并且也可以构造出许多新形式的五粒子最大纠缠态. 我们相信, 采用新的方法所构造的量子态, 将在进一步研究多粒子纠缠态方面起着非常重要的作用.

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A new method for constructing entangled states via orthogonal arrays

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A pure quantum state of N subsystems with d levels each is called k -multipartite maximally entangled state, which we call a k -uniform state, if all its reductions to k qubits are maximally mixed. Recently, Goyeneche and Zyczkowski proposed a method of constructing k -uniform states via orthogonal arrays, by which, we will also propose a new method to construct quantum states with orthogonal arrays in this paper, which is the method of increasing phase parameters. We can find the quantum states constructed via our new method by using the same orthogonal array, for four-qubit quantum states, not only are they 1-uniform states, also may become maximally entangled states; for five-qubit quantum states, we find that it can make the original 1-uniform states into 2-uniform states. In particular, for four-qubit quantum states, we can construct a series of new forms of maximally entangled states via our new method.

orthogonal arrays, k -uniform states, four-qubit entangled states

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