基于向量投影取样的改进加权响应面法

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摘要 在加权非线性响应面法的基础上, 通过向量梯度投影方法改进样本点的选择, 提出一种基于向量投影取样的改进加权响应面方法. 用向量梯度投影法获取新的试验样本点, 再对这些取样点进行加权回归处理, 赋予靠近极限状态曲面的样本点更高的权重来构造和更新迭代二次响应面函数, 解决隐式极限状态函数的结构可靠性分析问题. 该方法有效克服了传统响应面法计算结果受插值系数的影响, 一定程度上提高了计算精度, 可以得到相对更优的结果. 算例分析验证了所提方法的合理性和有效性.

关键词 响应面法, 加权响应面法, 极限状态函数, 结构可靠性

在进行结构可靠性分析时, 如果极限状态函数(Limit State Function, LSF)是显式表达, 积分法、一次二阶矩法等是较简单、常用的解答方法. 但实际结构的可靠性分析问题, 大多属于隐式极限状态函数的情况. 响应面法(Response Surface Method, RSM)的提出为解决隐式极限状态函数的可靠性分析问题提供了一种有效方法. 响应面法原理简单^[1]: 通过一系列确定性实验, 用多项式函数来近似隐式极限状态函数. 通过合理地选取试验点和迭代策略, 保证多项式函数能够在失效概率上收敛于真实的隐式极限状态函数.

由于RSM易于操作, 获得了广泛的关注和应用^[2~6]. 1990年, Bucher等人^[7]采用不含交叉项的二次多项式函数近似结构的极限状态函数, 将响应面法用于结构的可靠性分析. 1993年, Rajashekhar等人^[8]进一步发展了响应面法, 经自适应迭代推广了响应面法. Kim等人^[9]提出了改进取样点的向量梯度投影响应面法(Vector Projection Response Surface Method,

VPRSM). 在此基础上, 王宇等人^[10]把向量投影响应面法引入到边坡可靠性评价中, 改进了对非线性程度较高的极限状态函数求解可靠指标的收敛性. 但该方法未考虑产生的试验样本点距离真实极限状态函数的远近, 把试验点的作用看成了同等的效果. 2005年, Kaymaz等人^[11]提出了加权响应面法(Weighted Response Surface Method, WRSM)的思想, 在明显减小工作量的前提下提高了响应面法的精度, 但加权回归处理的试验点代表性不足, 试验点的选择策略还可以优化改进. 随着结构可靠性分析理论的发展, 一些研究者^[12~14]进一步改进了加权响应面法. 范文亮等人^[15]提出了考虑交叉项的自适应响应面法, 赵威等人^[16]使用偏最小二乘回归分析替代最小二乘法实现响应面法性能的提高. 这些思想进一步丰富了结构可靠性问题的求解方法. 但一些方法可能会发生抽样点的组合爆炸, 或可能因不合理的插值系数产生较大误差, 甚至导致错误的结果^[17].

针对上述问题, 本文在加权非线性响应面方法

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的基础上,提出了一种基于向量投影取样的改进加权响应面方法.选择响应面的形式和选择样本点的方式是影响计算结果精度和效率的关键^[18,19],这也是改进响应面法的主要方向和思路.本文方法是在经过设计点的向量切线上选择试验点,并利用试验点对极限状态函数的近似程度选择加权试验点,以拟合出极限状态函数,计算可靠性指标.将选择出来的试验点投影到上一次的响应面上,再对试验点中起作用大的样本点增加权重,用这些点的极限状态函数值构成的权函数度量试验点的作用程度.所提方法提高了对极限状态函数的近似精度,而且提高了计算可靠性指标的稳定性,减弱了插值系数变化对响应面结果的影响.

1 向量梯度投影和加权系数的构造

在结构可靠性分析中,取响应面函数的形式为

$$\bar{g}(x) = a + \sum_{i=1}^n b_i x_i + \sum_{i=1}^n c_i x_i^2, \quad (1)$$

其中 $a, b_i, c_i (i=1, 2, \dots, n)$ 为 $2n+1$ 个(n 是变量个数)待定系数.确定这些待定系数,至少需要 $2n+1$ 个试验点.一般采用中心复合设计法选取试验样本点,坐标轴上取样点坐标为 $x_i = u_{xi} \pm f \sigma_{xi}$.其中, x_i 为试验点坐标, u_{xi} 为其均值, σ_{xi} 为其标准差, f 为插值系数,常取经验值1~3. f 确定了试验点的取值范围,不同的 f 对计算结果的影响不同,有时 f 值会使结果不收敛.因此,如何减少 f 对可靠性分析结果的影响是响应面法需要研究的问题.

考虑改变样本点的取样方式,通过向量梯度投影取样优化传统抽样方式,可以使响应面能够在设计点附近更好地拟合真实极限状态曲面.向量梯度投影法的实现过程如下.

第一步,计算响应面函数在验算点处的单位列向量^[10]:

$$\omega = -\frac{\nabla \bar{g}(x^*)}{\|\nabla \bar{g}(x^*)\|}, \quad (2)$$

其中 $\nabla \bar{g}(x^*)$ 为 $g(x^*)$ 的梯度向量, x^* 为上次迭代得到设计点, ω 为与极限状态函数的切线垂直且指向极限状态函数方向的向量.

第二步,计算第 i 个随机变量 x_i 的取样点投影所用的单位向量 t_i ^[10],即为

$$t_i = \begin{cases} \frac{T_i}{\|T_i\|}, & T_i \neq 0, \\ 0, & T_i = 0, \end{cases} \quad (3)$$

$$T_i = e_i - (\omega^T e_i) \omega, \quad (i=1, 2, \dots, n), \quad (4)$$

其中, $e_i = (\delta_{1i}, \delta_{2i}, \dots, \delta_{ni})^T$ 为沿着 x_i 坐标轴的单位基向量.

$$\delta_{ij} = \begin{cases} 1, & (i=j), \\ 0, & (i \neq j). \end{cases} \quad (5)$$

以二维变量取样为例,展示设计点 x^* 样本选取情况,如图1所示.

每一次迭代得到的设计点向上次迭代得到的响应面上进行投影,可以使所有的试验点较合理地分布于真实极限状态曲面附近.可以得到单位投影向量 $q_i (i=1, 2, \dots, n)$ ^[10]为

$$\begin{cases} q_i = \frac{Q_i}{\|Q_i\|}, \\ Q_i = \tau(t_i + \varepsilon \omega) + (1 - \tau)e_i, \end{cases} \quad (6)$$

其中: ε 为一小数; $0 \leq \tau \leq 1$ 是加权因子, $\tau=1$ 表示完全向量梯度投影, $\tau=0$ 为中心复合设计法.根据式(2)所确定的投影方向取样,取样点的坐标变为^[20]

$$x_i = u_{xi} \pm f \sigma_{xi} q_i^T e_i, \quad (i=1, 2, \dots, n). \quad (7)$$

这样就得到了向量投影取样的 m 个取样点,一般可以取 $m=2n+1$ 个点,来确定响应面的多项式函数,采用最小二乘法求解待定系数 B ^[20]:

$$B = (A^T A)^{-1} A^T y, \quad (8)$$

$$A = \begin{bmatrix} 1 & x_{11} & \cdots & x_{1n} & x_{11}^2 & \cdots & x_{1n}^2 \\ 1 & x_{21} & \cdots & x_{2n} & x_{21}^2 & \cdots & x_{2n}^2 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ 1 & x_{m1} & \cdots & x_{mn} & x_{m1}^2 & \cdots & x_{mn}^2 \end{bmatrix}, \quad (9)$$

其中, $y = (g(x_1), g(x_2), \dots, g(x_m))^T$ 为试验点对应的响应量列阵, A 为试验点构成的回归系数矩阵.

图1 向量投影取样示意图

Figure 1 Vector projection sampling points

在响应面中,关键问题是如何用 $\bar{g}(x)=0$ 高精度地拟合 $g(x)=0$,试验点距离真实极限状态曲面有远有近,它们在确定响应面时所起的作用也就有大有小,更靠近极限状态曲面的样本点应起到更大作用,若将这些试验点同等对待,就没有体现出各个试验点所发挥的不同重要程度,那么会降低响应面的拟合精度.加权响应面法通过对试验点合理赋权,使 $|g(x_i)|$ 较小的试验点在确定隐式极限状态函数时起到更加重要的作用,实现 $\bar{g}(x)=0$ 更好地逼近于 $g(x)=0$.试验点的权重决定了该试验点在回归分析中的作用.发挥作用越大,相对越重要的点,应该被赋予更大的权重.用试验点距离极限状态曲面 $g(x)=0$ 的远近程度,确定构造权重系数 $W^{[21,22]}$.可以通过式(10)和(11)构造权重系数,赋予 $|g(x_i)|$ 函数值较小的试验点以较大的权重系数.

$$\begin{cases} g_{\text{best}} = \min \{|g(x_1)|, |g(x_2)|, \dots, |g(x_n)|\}, \\ \omega_i = \frac{g_{\text{best}}}{|g(x_i)|}, \end{cases} \quad (i=1, 2, \dots, n), \quad (10)$$

$$W = \begin{bmatrix} \omega_1 & 0 & 0 & 0 \\ 0 & \omega_2 & 0 & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & \omega_n \end{bmatrix}. \quad (11)$$

2 向量投影取样的改进加权响应面法

将向量梯度投影法与加权非线性响应面法结合,利用向量投影法获取性能更优试验点的能力和加权非线性响应面法的加权回归寻优能力,可以在避免两者缺点的同时较好地优势互补.将试验拟合点向上次迭代的响应面上做投影,优化改进了样本点的选择策略,试验点会更合理地分布于真实极限状态曲面附近.通过进一步分析试验点的重要程度,赋予试验点相应的权重系数,这样对于真实极限状态曲面附近的试验点,又进行了一次筛查,就不会浪费已有试验点的有用信息,避免引入劣质试验点,充分考虑了不同试验点的作用大小.经过投影和加权处理后的试验点随响应面不断地向真实极限状态曲面靠近,最终收敛于真实极限状态曲面.此时,改进样本点的选择策略,选择出来的试验点代表性增强.对试验点进行加权处理分析,优化了试验点的权重.算法的稳定性更好,效率更高,稳健性更强.

基于向量梯度投影取样的加权响应面法的主要

思想是:用向量投影法获取试验点,对试验点进行加权回归分析,更新响应面进行迭代计算,结果收敛时,即可得到可靠性指标.其具体方法的实现步骤如下:

第一步:取初始迭代点 $x_d^{(1)} = (x_{d1}^{(1)}, x_{d2}^{(1)}, \dots, x_{dm}^{(1)})^T$.一般可采用均值点 u_k ,采用经典响应面法确定 $\bar{g}^{(1)}(x)$.

第二步:第 $k(k \geq 2)$ 次迭代,以线性插值得到的试验点 $x_1^{*(k)}$ 为抽样中心点,并围绕 $x_1^{*(k)}$ 选取新增试验点 $(x_{11}^{*(k)}, x_{12}^{*(k)}, \dots, x_{li}^{*(k)} \pm f^{(k)}\sigma_{xi}, \dots, x_{ln}^{*(k)})$,并定义 f 值,加上 $x_1^{*(k)}$ 共有 $2n+1$ 个试验点,它们共同构成了第 k 次加权最小二乘回归分析的试验点.

第三步:将试验点代入式(9),得到系数矩阵 A ,计算极限状态函数的估计值 $\bar{g}_i(x)$ ($i=1, 2, \dots, n, \dots, 2n+1$).对于复杂结构一般需使用有限元方法或其他数值方法.

第四步:构造加权矩阵.进行回归分析,计算试验点处向量 y ,并由式(11)构成权重矩阵 W ,用加权最小二乘法确定二次多项式第 k 次迭代的待定系数矩阵.

$$B = (A^T W A)^{-1} A^T W y, \quad (12)$$

来代替式(8),增加系数矩阵的权重.

第五步:求出第 k 次迭代的响应面函数的设计计算点 $x^{*(k)}$ 和可靠性指标 $\beta^{(k)}$.

第六步:用均值点 $(u_x, g(u_x))$ 和上一步得到的设计点 $(x_d^{(k)}, g(x_d^{(k)}))$ 进行线性插值,求得使 $g(x_1^{*(k+1)}) \approx 0$ 的下次迭代抽样中心 $x_1^{*(k+1)}$,其中 $x_1^{*(k+1)}$ 坐标为^[22]

$$x_{li}^{*(k+1)} = \mu_{xi} + (x_{Di}^{(k)} - \mu_{xi}) \frac{g(\mu_x)}{g(\mu_x) - g(x_{Di}^{(k)})}. \quad (13)$$

第七步:计算响应面函数在 $x_1^{*(k+1)}$ 处的单位向量 ω ,由式(7)计算取样点坐标,同时 f 取开方值.计算出下一步的试验点坐标.

第八步:判断前后两次迭代计算得到的设计点的相对误差是否满足要求.如果精度不足,重复第二到第七步,直到满足精度要求 $\|x_D^{(k)} - x_D^{(k-1)}\| < \varepsilon$ 完成计算.

以上过程的实现流程如图2所示.

3 算例分析

算例1:考虑如下指数型极限状态函数: $g(x) = \exp(0.2x_1 + 1.4) - x_2$.其中 x_1 和 x_2 为服从标准正态分布的随机变量.

图2 基于向量投影取样的改进响应面法流程图

Figure 2 Chart of the improved weighted response surface method based on vector projection sampling

以蒙特卡洛(Monte Carlo, MC)法的结果作为验证标准值. 经典响应面法、向量响应面法、加权非线性响应面法和本文方法的计算结果如表1和图3所示. 插值系数 f 分别取值2和8时, 由于取样点的变化范围非常大, RSM, WRS, VPRS的可靠性指标发生了变化, 误差也有所变大. 本文方法几乎不受插值系数变化的影响, 在较大的范围内较准确地拟合出极限状态函数曲线, 从而可有较高的精度.

算例2: 考虑图4门式框架, 计算失效概率. 各单元的弹性模量均为 $E=2 \times 10^6$ kN/m², 梁和柱的截面惯性矩为 $I_1=A_1^2/12$ 和 $I_2=A_2^2/6$. A_1 和 A_2 均服从对数正态分布, P 服从极值I型分布, 其均值和变异系数分别为 $u_{A_1}=0.36$ m², $V_{A_1}=0.1$, $u_{A_2}=0.18$ m², $V_{A_2}=0.1$, $u_P=20$ kN, $V_P=0.25$.

以节点3的水平位移 u_3 作为需控制的结构最大变形. 建立极限状态函数如下: $g=0.01-u_3(A_1, A_2, P)$. 其中, 水平位移 u_3 与随机变量 A_1 , A_2 , P 的关系不能显示表达, 此时为隐式极限状态函数的情况. 用Monte

表1 算例1 计算结果

Table 1 Calculation results of example 1

方法	f	β	$P_f(10^{-4})$	β 误差(%)
MC		3.3460	4.1000	
RSM	$f=2$	3.3499	4.0425	0.12
	$f=8$	3.3934	3.4515	1.42
WRS	$f=2$	3.3497	4.0442	0.11
	$f=8$	3.3499	4.0426	0.12
VPRS	$f=2$	3.3495	4.0484	0.10
	$f=8$	3.3907	3.4857	1.34
本文方法	$f=2$	3.3498	4.0442	0.11
	$f=8$	3.3498	4.0442	0.11

图3 算例1 极限状态函数图

Figure 3 The limit state function of sample 1

图4 门式框架

Figure 4 Model of portal frame

Carlo法计算, 调用 10^5 次有限元计算, 得到失效概率 $P_f=2.4085 \times 10^{-3}$. 利用本文方法调用有限元分析70次,

得到的失效概率 $P_f=2.3144\times 10^{-3}$, 相对误差为3.9070%. 最终拟合得到设计点坐标是(0.3157, 0.1696, 37.2562), 极限状态函数为

$$g = -0.0378 + 0.1716 \times A_1 + 0.1172 \times A_2 - 0.0003 \times P - 0.1959 \times A_1^2 - 0.2330 \times A_2^2.$$

算例3: 以旋转圆盘的破裂问题为例^[23]. 转盘破裂的极限状态函数: $M_b = \sqrt{\frac{FU}{L}} - M_a$. 其中, $L = \frac{\delta[2\pi v]^2(R_i^3 - R_o^3)}{3(385.82)(R_i - R_o)60^2}$. 各随机变量特性如表2所示. 当 M_a 取0.35~0.55时, 计算 M_b 的累积分布函数如图5所示.

图5表明, 不同响应面法所形成的多项式函数对极限状态函数的拟合不同. 本文方法计算出的转盘 M_b 累积分布函数与转盘Monte Carlo法累积分布函数拟合较一致, 差异较小, 拟合的精度较好. 文中方法得到的响应面函数为

$$g = 1.3976 - 0.0001 \times v - 0.0393 \times R_i - 0.0129 \times R_o - 1.9653 \times \delta + 0.3602 \times F + 0.0005 \times R_i^2 + 0.0001 \times R_o^2 + 2.0361 \times \delta^2 - 0.0634 \times F^2.$$

4 结论

本文针对隐式极限状态函数问题, 研究了结构可靠性分析的加权响应面法, 结合了向量投影法获取较优试验点和加权响应面法增加权重系数的优势, 提出了基于向量梯度投影取样的改进加权响应面方法. 所提方法可以较稳健地找到设计点, 得到较高精

表2 转盘随机变量参数

Table 2 Random variable parameters of disk

随机变量	分布	均值	标准差
U	正态	220000	5000
v	正态	21000	1000
R_o	正态	8	0.3
R_i	正态	24	0.5
δ	均匀	[0.28, 0.30]	
F	威布尔	[$b=25.508, h=0.958$]

图5 转盘破裂累积分布函数

Figure 5 Broken disk of cumulative distribution function

度的可靠性指标, 同时优化了传统响应面法计算结果易受人为因素影响的缺陷, 其稳定性和稳健性较好. 算例分析表明, 本文方法是有效的和可行的, 在一定程度上提高了响应面法的计算精度和适用范围.

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Summary for “基于向量投影取样的改进加权响应面法”

An improved weighted response surface method based on vector projection sampling

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Structural reliability theory stems from the nature of randomness, fuzziness, characteristic and some other uncertainties in the process of engineering structural design, construction and employment. With the rapid development of science technology and industry, many departments have realized the importance of structural reliability problem and its potential economic benefits. Solving the problem of structural reliability has quickly become an important issue in the field of academic research. Because of the complexity of the structure and the harsh working environment which lead to a complex structural reliability problem, the traditional quality analysis method can't explain the failure problems in practical engineering. How to solve the problem of large-scale complex structural reliability, improve the accuracy and efficiency of reliability analysis method, and further obtain great economic benefits, have become one of the most important exploration areas for the enterprises and scholars at home and abroad.

The response surface method, which can replace the implicit limit state function by small amount of computation arises at this time. Through a series of deterministic response surface method, it uses polynomial function to approximate implicit limit state function. By reasonably selecting sites and iteration strategy, it ensures that the polynomial function on the failure probability can converge to the failure probability of the implicit limit state function. Response surface method, with high efficiency of clarity and precision, strong operability, combined with finite element advantage, is a reliability method widely used at present. In the process of the implementation of the response surface method, it uses three key steps of selecting the response surface function forms, obtaining sample points by the experiment design and using the regression fitting model, which has direct impact on the degree of the response surface method approximating limit state function and determines the performance of the response surface method. This is a problem that the response surface method must solve. It is the paper's original intention to conduct the research work based on the response surface method of structural reliability optimization, improving the above three steps and the efficiency of engineering structure. The paper aims at the implicit limit state function problem, studying the structural reliability analysis of the weighted response surface method, combining the advantage of obtaining better sites by vector projection method and increases weighted coefficient by weighted response surface method. The improved weighted response surface method based on vector projection sampling is proposed. It uses the vector of the gradient projection method to get new design point and sample points, giving the actual limit state function of sample points more weights to construct the quadratic response surface function, updating the iterative response surface function, and solves the problem of the structural reliability of the implicit limit state function.

Example analysis shows the characteristics of the proposed method, a steady design point can be found, and the calculation of stability has been improved considerably. The classical response surface method is optimized and reduces the defects that the calculation results are seriously affected by the interpolation coefficient. To some extent, it improves the calculation accuracy and can get relatively better results. Improved weighted response surface method based on vector projection sampling combines the advantages of obtaining test sample points by vector projection and weighted regression method, which is effective, feasible and can operate directly. It extends the application field of the response surface method to some degree and gets better approximate fitting limit state function at design point. It has better stability and robustness and provides new approach and ideas to solve the implicit limit state function.

response surface method, weighted response surface method, limit state function, structural reliability

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