预应力作用下双层石墨烯低温热振动

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摘要 热振动对纳尺度器件动力学特性有着重要的影响. 为此, 建立了预应力作用下, 考虑量子效应的双层矩形石墨烯的等效连续介质模型. 计算了不同预应力作用下, 不同温度时双层石墨烯的固有频率和均方根振幅. 双层石墨烯层间存在范德华相互作用, 其同向振动固有频率与相同尺寸单层石墨烯固有频率相等, 双层石墨烯存在大于同向振动频率的反向振动频率. 考虑量子效应板模型得到的均方根振幅值小于结合能量均分原理板模型得到的结果. 且随着温度的降低, 两种板模型结果之间的差值增大, 量子效应越来越明显. 随着预应力的增加, 双层石墨烯的固有频率值增加, 均方根振幅值降低, 量子效应的影响逐渐增加. 预应力对同向振动的影响远大于对反向振动的影响. 分析了零点能量对双层石墨烯热振动的影响, 当计入零点能量后, 石墨烯均方根振幅大于结合能量均分原理板模型预测的结果, 且在零温时依然存在不为零的振幅.

关键词 双层石墨烯, 热振动, 范德华力, 量子效应, 板模型, 零点能量

热振动是非零温下物体的固有属性, 对于微尺度, 尤其是纳尺度传感器、振动元件等的动力学特性有着重要的影响^[1~6]. 原子热振动引起的热噪声是力学测量精度的根本极限, 对于高灵敏度的微尺度器件, 热噪声引起的测量误差已经引起了广泛的研究^[7]. 由能量均分原理可知, 在热平衡状态下, 系统每一个自由度动能的平均值等于 $k_B T/2$, 其中 k_B 为波尔兹曼常数, T 为系统温度. 基于能量均分原理, 结合梁理论, 可计算得到热噪声引起的小尺寸悬臂梁结构的振幅^[8]. 当结构的尺寸进入纳尺度, 系统的热振动幅值相对于结构本身尺寸来说更为明显. 因此在纳尺度结构动力学问题的研究中, 热振动问题是不可忽略的一部分. Treacy等人^[9]和Krishnan等人^[10]采用透射电子显微镜测量了碳纳米管的热振动振幅, 并结合能量均分原理得到了碳纳米管的等效杨氏模

量. 另外, 利用梁理论模型结合能量均分原理可较准确地预测热平衡状态下碳纳米管的均方根振幅^[11~13]. 能量均分原理在 $\hbar\omega \ll k_B T$ 条件下具有广泛的适用性, 其中 $\hbar$ 为普朗克常数, ω 为频率. 但当 $\hbar\omega$ 的值接近甚至超过 $k_B T$ 时, 根据普朗克定理, 能量均分原理不再成立, 量子效应逐渐明显. 而对于工作在较低温度的纳尺度器件, 由于其固有频率可达到太赫兹量级, 能量均分原理成立所要求的 $\hbar\omega \ll k_B T$ 不能满足, 因此在进行此类结构的热振动问题研究时需计入量子效应的影响. Wang和Hu^[14]采用考虑量子效应的梁模型得到了单壁碳纳米管热振动的均方根振幅, 发现低温下, 考虑量子效应时的均方根振幅明显小于经典假设下的结果. Liu和Wang^[15]采用半量子分子动力学方法得到了单壁碳纳米管的热振动谱和均方根振幅, 并与含量子效应的梁模型结果进行了对比.

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二维纳米材料石墨烯因其出色的力、电、光及热等性质受到了大量的关注^[16~18]. 多篇文献^[19~22]采用分子动力学方法和等效连续介质板模型, 研究了经典条件下石墨烯的热振动问题. Wang和Hu^[23,24]采用考虑量子效应的板模型研究了单层石墨烯热振动的均方根振幅. 而对于多层石墨烯, 其层间存在范德华力的作用, 表现出了更复杂的动力学性质^[25,26]. Lee和Chang^[27]研究了考虑量子效应的圆形多层石墨烯的热振动问题. 然而, 对于存在层间范德华力的矩形双层石墨烯热振动中量子效应影响的研究, 尚未见报道. Bunch等人^[28]和Lee等人^[29]曾指出, 在实验条件下石墨烯会受到基底分子的范德华力作用, 内部常呈现受拉的状态, 并且这种与基底的相互作用是不可避免的. 因此需要研究预应力对石墨烯热振动的影响. 本文建立了考虑量子效应和预应力作用的双层石墨烯等效板模型, 计算其固有频率及不同温度下的均方根振幅, 定量分析了量子效应对受不同预应力、处于不同温度下的双层石墨烯热振动的影响.

1 双层石墨烯等效板模型

双层石墨烯层与层之间存在范德华力, 若石墨烯做小幅振动, 可将范德华力等效为刚度不变的线性弹簧连接. 每层石墨烯可等效为各向同性的弹性薄板, 如图1所示, 并引入非局部效应. 假设石墨烯长宽分别沿直角坐标系的 x , y 方向, 因此考虑层间范德华力双层板的动力学方程

$$D\nabla^4 w_1 + r^2 D\nabla^6 w_1 + \rho h_g \frac{\partial^2 w_1}{\partial t^2} = c_{12}(w_1 - w_2) + N_x \frac{\partial^2 w_1}{\partial x^2} + 2N_{xy} \frac{\partial^2 w_1}{\partial x \partial y} + N_y \frac{\partial^2 w_1}{\partial y^2}, \quad (1a)$$

$$D\nabla^4 w_2 + r^2 D\nabla^6 w_2 + \rho h_g \frac{\partial^2 w_2}{\partial t^2} = c_{12}(w_2 - w_1) + N_x \frac{\partial^2 w_2}{\partial x^2} + 2N_{xy} \frac{\partial^2 w_2}{\partial x \partial y} + N_y \frac{\partial^2 w_2}{\partial y^2}, \quad (1b)$$

其中 $\nabla^4 w = \frac{\partial^4 w}{\partial x^4} + 2\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4}$, $\nabla^6 w = \frac{\partial^6 w}{\partial x^6} + 3\frac{\partial^6 w}{\partial x^4 \partial y^2} + 3\frac{\partial^6 w}{\partial x^2 \partial y^4} + \frac{\partial^6 w}{\partial y^6}$. $D = Eh_g/12(1-\mu^2)$ 为石墨烯弯曲刚度, 其中 E , h_g , μ 分别为等效杨氏模量、厚度和泊松比. N_x , N_y 分别为板沿 x , y 方向单位长度所受

图1 (网络版彩色) 双层石墨烯等效双层板模型

Figure 1 (Color online) The equivalent double plates model of the double-layered graphene

的力, 本文计算中对板 x 和 y 两个方向施加相同的应变 ε , 此时预应力为: $N_x = N_y = E\varepsilon h_g$, $N_{xy} = 0$. c_{12} 为范德华相互作用系数. w_1, w_2 分别为板1和板2沿 z 方向的挠度. r 为非局部参数

$$r = \frac{d}{\sqrt{12}}, \quad (2)$$

d 为原子间距离, 对于石墨烯, $d=0.142 \text{ nm}$ ^[30].

当上下两层石墨烯的边界条件均为四边简支时, 可假设两板的挠度分别为

$$w_{1-mn} = A_{mn} W_{mn}(x, y) e^{j\omega_{mn}t} = A_{mn} \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b} e^{j\omega_{mn}t}, \quad (3a)$$

$$w_{2-mn} = B_{mn} W_{mn}(x, y) e^{j\omega_{mn}t} = B_{mn} \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b} e^{j\omega_{mn}t}, \quad (3b)$$

其中 $m, n=1, 2, \dots$, A_{mn} , B_{mn} 为待定系数, a , b 分别为板的长度和宽度, $j \equiv \sqrt{-1}$, ω_{mn} 为固有频率. 将式(3)代入式(1)得

$$DA_{mn}\pi^4 \left(\frac{m^2}{a^2} + \frac{n^2}{b^2} \right)^2 R - \rho h_g A_{mn} \omega_{mn}^2 = c_{12}(A_{mn} - B_{mn}) - A_{mn} \left(N_x \frac{m^2 \pi^2}{a^2} + N_y \frac{n^2 \pi^2}{b^2} \right), \quad (4a)$$

$$DB_{mn}\pi^4 \left(\frac{m^2}{a^2} + \frac{n^2}{b^2} \right)^2 R - \rho h_g B_{mn} \omega_{mn}^2 = c_{12}(B_{mn} - A_{mn}) - B_{mn} \left(N_x \frac{m^2 \pi^2}{a^2} + N_y \frac{n^2 \pi^2}{b^2} \right), \quad (4b)$$

其中 $R = \left[1 - r^2 \pi^2 \left(\frac{m^2}{a^2} + \frac{n^2}{b^2} \right) \right]$, 消去方程组中的系数 A_{mn} 和 B_{mn} 得

$$\left[D\pi^4 \left(\frac{m^2}{a^2} + \frac{n^2}{b^2} \right)^2 R - \rho h_g \omega_{mn}^2 + \left(N_x \frac{m^2 \pi^2}{a^2} + N_y \frac{n^2 \pi^2}{b^2} \right) - c_{12} \right]^2 = (c_{12})^2, \quad (5)$$

因此可得两支振动频率分别为

$$\omega_{l-mn} = \sqrt{\frac{D\pi^4 \left(\frac{m^2}{a^2} + \frac{n^2}{b^2} \right)^2 R + N_x \frac{m^2 \pi^2}{a^2} + N_y \frac{n^2 \pi^2}{b^2}}{\rho h_g}}, \quad (6a)$$

$$\omega_{A-mn} = \sqrt{\frac{D\pi^4 \left(\frac{m^2}{a^2} + \frac{n^2}{b^2} \right)^2 R + N_x \frac{m^2 \pi^2}{a^2} + N_y \frac{n^2 \pi^2}{b^2} - 2c_{12}}{\rho h_g}}, \quad (6b)$$

将式(6a)代入式(4)可得 $A_{mn}/B_{mn}=1$, 而将式(6b)代入式(4)可得 $A_{mn}/B_{mn}=-1$. 从上式可以看出, 第一支振动频率式(6a), 与单层板振动频率表达式完全相同, 代表双层板的同向振动频率. 第二支频率式(6b), 代表反向振动频率, 相同阶数时大于同向振动频率.

在板的谐振过程中, 对于某一阶振动模式, 势能和动能不停地相互转化, 且最大势能值等于最大动能值, 分别等于此阶模式的总能量. 因此可计算板通过平衡位置时的动能, 得到双层板 mn 阶总能量, 此时势能为零

$$\begin{aligned} \langle E_{mn} \rangle &= 2 \times \frac{\rho h_g}{2} \int_0^a \int_0^b \left(\frac{\partial w_{mn}(x, y, t)}{\partial t} \right)^2 dx dy \\ &= \rho h_g \int_0^a \int_0^b \left(A_{mn} \omega_{mn} \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b} \right)^2 dx dy \\ &= \frac{\rho h_g A_{mn}^2 \omega_{mn}^2 ab}{4}. \end{aligned} \quad (7)$$

当石墨烯处于热平衡状态时, 在经典条件下, 根据能量均分原理, 各阶模式能量相等, 均符合 $\langle E_{mn} \rangle = k_B T$, 因此可得到挠度函数中的系数为

$$\begin{aligned} A_{l-mn}^2 &= 4k_B T / \rho h_g \omega_{l-mn}^2 ab \\ &= \frac{4k_B T}{\left[D\pi^4 ab \left(\frac{m^2}{a^2} + \frac{n^2}{b^2} \right)^2 R + N_x ab \frac{m^2 \pi^2}{a^2} + N_y ab \frac{n^2 \pi^2}{b^2} \right]}, \end{aligned} \quad (8)$$

而对于反向振动, 同样可求得挠度函数中的系数为

$$\begin{aligned} A_{A-mn}^2 &= 4k_B T / \rho h_g \omega_{A-mn}^2 ab \\ &= \frac{4k_B T}{\left[D\pi^4 ab \left(\frac{m^2}{a^2} + \frac{n^2}{b^2} \right)^2 R + N_x ab \frac{m^2 \pi^2}{a^2} + N_y ab \frac{n^2 \pi^2}{b^2} - 2abc_{12} \right]}. \end{aligned} \quad (9)$$

当能量均分原理不成立, 即 $\hbar \omega_{mn}$ 接近或大于 $k_B T$ 时, 令 $\langle E_{mn} \rangle = \frac{\hbar \omega_{mn}}{e^{\hbar \omega_{mn}/k_B T} - 1}$, 可得包含量子效应时挠度函数中的系数为

$$\begin{aligned} A_{l-mn}^2 &= \frac{4\hbar \omega_{l-mn} / e^{\hbar \omega_{l-mn}/k_B T} - 1}{\rho h_g \omega_{l-mn}^2 ab} \\ &= \frac{4\hbar \omega_{l-mn} / e^{\hbar \omega_{l-mn}/k_B T} - 1}{\left[D\pi^4 ab \left(\frac{m^2}{a^2} + \frac{n^2}{b^2} \right)^2 R + N_x ab \frac{m^2 \pi^2}{a^2} + N_y ab \frac{n^2 \pi^2}{b^2} \right]}, \end{aligned} \quad (10)$$

反向振动时

$$\begin{aligned} A_{A-mn}^2 &= \frac{4\hbar \omega_{A-mn} / e^{\hbar \omega_{A-mn}/k_B T} - 1}{\rho h_g \omega_{A-mn}^2 ab} \\ &= \frac{4\hbar \omega_{A-mn} / e^{\hbar \omega_{A-mn}/k_B T} - 1}{\left[D\pi^4 ab \left(\frac{m^2}{a^2} + \frac{n^2}{b^2} \right)^2 R + N_x ab \frac{m^2 \pi^2}{a^2} + N_y ab \frac{n^2 \pi^2}{b^2} - 2abc_{12} \right]}. \end{aligned} \quad (11)$$

进一步可得各阶模式的均方根振幅值

$$w_{mn}^{RMS} = \frac{\sqrt{2}}{2} \left| A_{mn} \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b} \right|, \quad (12)$$

同向振动时 $A_{mn} = A_{l-mn}$, 反向振动时 $A_{mn} = A_{A-mn}$.

2 结果与讨论

为了进行定量计算, 需要确定石墨烯的等效弹性模量 E 、泊松比 μ 和等效厚度 h_g . 本文中采用的参数为 $\mu=0.41$, $h_g=0.089$ nm, 刚度 $K=Eh_g=245$ N/m^[20]. 另外, 取 $a=12$ nm, $b=10$ nm 的双层石墨烯作为研究对象, 计算中取范德华相互作用系数 $c_{12}=-108 \times 10^{18}$ Pa/m^[31].

首先, 计算了双层石墨烯的固有频率, 图2给出了零初始应力作用时双层石墨烯同向振动和反向振

图2 (网络版彩色) 双层石墨烯同向振动和反向振动前10阶固有频率. 图中方点表示同向振动频率, 圆点表示反向振动频率

Figure 2 (Color online) The first ten natural frequencies of in-phase vibration (circle dot) and anti-phase vibration (square) of the double-layered graphene

动的前10阶固有频率. 由式(6)并结合图2可以看出, 双层石墨烯同向振动频率值与相同长、宽的单层石墨烯的固有频率完全相同. 所不同的是, 双层石墨烯具有1支反向振动频率, 且由于范德华系数 c_{12} 值较大, 因此反向振动频率远大于同向振动频率.

图3为不同预应力作用下, 双层石墨烯同向振动和反向振动第一阶频率的变化. 可以看到, 随着预应力的增加, 石墨烯的固有频率值增加, 且同向振动频率值增速要大于反向振动频率增速. 这说明预应力的存在对同向振动频率的影响要大于对反向振动频率的影响.

图4列出了双层石墨烯同向振动和反向振动的前3阶振动模式示意图. 可以看出, 对应相同的阶数, 同向振动和反向振动拥有相同的振型, 所不同的是, 反向振动中, 两层石墨烯朝相反的方向运动. 为了方便定量地研究石墨烯热振动过程的振幅情况, 取沿长度方向板中间位置上的点绘出同向振动和反向振动的均方根振幅, 如图5所示. 因为双层石墨烯每层的均方根振幅值完全相同, 因此图中只表示出了其中1层板的结果. 当温度为50 K, 长宽两个方向应变

均为0.02时, 对于同向振动前两阶模态, 考虑量子效应板模型结果与经典板模型结果比较接近. 由于双层石墨烯同向振动频率与相同尺寸单层石墨烯固有频率值相等, 且热平衡状态下, 双层石墨烯各阶总能量与单层石墨烯各阶总能量相等, 因此单层石墨烯

图3 (网络版彩色)不同应变下双层石墨烯同向振动和反向振动基频变化曲线. 图中方点表示同向振动频率, 圆点表示反向振动频率

Figure 3 (Color online) The frequency of in-phase vibration (circle dot) and the frequency of anti-phase vibration (square) versus strain

图4 (网络版彩色)双层石墨烯同向振动和反向振动前3阶振型示意图. (a) 同向振动第一阶振动模式; (b) 同向振动第二阶振动模式; (c) 同向振动第三阶振动模式; (d) 反向振动第一阶模式; (e) 反向振动第二阶模式; (f) 反向振动第三阶模式

Figure 4 (Color online) The first three in-phase vibration and anti-phase vibration modes of double-layered graphene. (a) The first in-phase vibration mode; (b) the second in-phase vibration mode; (c) the third in-phase vibration mode; (d) the first anti-phase vibration mode; (e) the second anti-phase vibration mode; (f) the third anti-phase vibration mode

图5 (网络版彩色)应变为 0.02, 温度为 50 K 时由经典板模型和含量子效应板模型计算得到的双层石墨烯同向振动(a)和反向振动(b)的前两阶均方根振幅值

Figure 5 (Color online) The RMS amplitude of the first two in-phase vibration (a) and anti-phase vibration (b) of double-layered graphene obtained by the classical plate models and the plate models considering quantum effects at $T=50$ K, $\varepsilon=0.02$

热振动均方根振幅值为双层石墨烯均方根振幅的 $\sqrt{2}$ 倍。而对于反向振动前两阶模态, 考虑量子效应板模型得到的均方根振幅明显小于经典板模型结果。与同向振动不同的是, 由于反向振动前几阶频率非常接近, 因此前两阶均方根振幅最大值相差不大。另外, 从图中给出的定量数值可以看出, 温度较低时双层石墨烯反向振动均方根振幅值非常小, 因此本文所选取的定常范德华系数是适用的。

由式(10)可知, 当石墨烯处于不同环境温度时, 量子效应的影响不同。图6为不同温度时由两种板模型计算得到的石墨烯中点位置第一阶均方根振幅随温度的变化, 计算中仍假设石墨烯处于应变为0.02的双向拉伸状态。随着温度的降低, 经典板模型和考虑量子效应板模型得到的均方根振幅值之间的差值越来越大, 说明温度越低时, 量子效应越明显。而由于反向振动频率大于同向振动频率, 因此两种板模型

得到的反向振动均方根振幅相对差值要大于同向振动均方根振幅相对差值, 且在室温(300 K)时, 这一差值仍不可忽略。

分析了预应力对石墨烯热振动均方根振幅的影响, 取热平衡温度为50 K。图7为不同预应力作用下由两种板模型得到板中点位置的第一阶均方根振幅。由图7(a)可以看到, 随着预应力的增加, 均方根振幅值在减小, 且因为第一阶固有频率在增大, 因此两种板模型之间的差值在增加。说明预应力越大, 量子效应越明显。而对于反向振动, 在一定范围内的预应力变化对频率的影响很有限, 因此反向振动第一阶均方根振幅值在不同预应力作用下几乎没有变化。

在上述研究中并未考虑零点能量对热振动振幅的影响。而关于零点能量是否真实存在仍然存在争议^[32], 有实验表明了零点能量的存在^[33], 但也有学者认为零点能量是由于测不准原理造成的实验现

图6 (网络版彩色)应变为 0.02 时, 由经典板模型和含量子效应板模型计算得到的双层石墨烯中点位置同向振动(a)和反向振动(b)均方根振幅值随温度的变化趋势

Figure 6 (Color online) The RMS amplitude of in-phase vibration (a) and anti-phase vibration (b) of double-layered graphene at the middle point obtained by the classical plate models and the plate models considering quantum effects versus the temperature, $\varepsilon=0.02$

图7 (网络版彩色)温度为 50 K时,由经典板模型和含量子效应板模型计算得到的双层石墨烯中点位置同向振动(a)和反向振动(b)均方根振幅值随应变的变化趋势

Figure 7 (Color online) The RMS amplitude of in-phase vibration (a) and anti-phase vibration (b) of double-layered graphene at the middle point obtained by the classical plate models and the plate models considering quantum effects versus the strain, $T=50$ K

象^[34,35]. 计入零点能量后,各阶模态的平均能量为

$$\langle E_{mn} \rangle = \frac{1}{2} \hbar \omega_{mn} + \frac{\hbar \omega_{mn}}{e^{\hbar \omega_{mn} / k_B T} - 1},$$

图8为不同温度时考虑量子效应板模型和经典板模型计算得到的同向振动和反向振动第一阶均方根振幅. 当计入零点能量后,考虑量子效应板模型均方根振幅大于结合能量均分原理板模型结果,随着温度的降低,两种板结果之间的差值增加,且当温度为0 K时,经典板模型得到的振幅为0,而考虑量子效应板模型存在一个大于0的振幅. 从上面的计算结果可以看出,是否计入零点能量对均方根振幅的影响非常大,因此关于零点能量对热振动的作用仍需要进一步的理论和实验方面的研究.

3 总结

以矩形双层石墨烯为研究对象,考虑层间范德华力作用、预应力作用及非局部效应,建立了等效连续介质板模型,结合经量子修正的热平衡理论,分析了

量子效应对双层石墨烯热振动的影响,并与结合能量均分原理的板模型结果进行了对比. 结果显示,双层石墨烯振动模态中存在同向振动和反向振动,同向固有频率与相同尺寸单层石墨烯固有频率相等,反向振动频率值远大于同向振动频率值. 考虑量子效应板模型得到的均方根振幅小于结合能量均分原理板模型得到的结果,这一现象对于频率值更高的反向振动模态更为明显. 两种板模型结果之间的差值随着温度的降低而增大,说明量子效应越来越明显. 双层石墨烯的固有频率值随着预应力的增加而增加,相对应的均方根振幅值降低,量子效应的影响逐渐增加. 预应力对同向振动的影响远大于对反向振动的影响. 当在量子化模态能量中考虑零点能量后,板模型计算得到的均方根振幅值相对于未考虑零点能量板模型结果明显增加,且在很大范围内大于结合能量均分板模型所得均方根振幅. 随着温度的降低,零点能量的影响更为明显,且在零温时石墨烯仍存在不为零的振幅.

图8 (网络版彩色)应变为 0.02 时,由经典板模型和含量子效应板模型(计入零点能量)计算得到的双层石墨烯中点位置同向振动(a)和反向振动(b)均方根振幅值随温度的变化趋势

Figure 8 (Color online) The RMS amplitude of in-phase vibration (a) and anti-phase vibration (b) of double-layered graphene at the middle point obtained by the classical plate models and the plate models considering quantum effects (including zero-point energy) versus the temperature, $\varepsilon=0.02$

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Summary for “预应力作用下双层石墨烯低温热振动”

Thermal vibration of a double-layered graphene sheet with initial stress at low temperature

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Thermal fluctuations play an important role in the resonance properties of low dimensional structures as they serve as acoustic and vibration sensors of micro and nano machine. The quantum effects will be obvious during the thermal vibration of nanostructures because of the high natural frequency. Graphene has novel electronic properties and superior mechanical strength. The thermal vibration problems of graphene, which can be used as nano electronic components, have attracted a lot of research interest. The influence of quantum effects for the thermal vibration of single-layered graphene has been investigated. The dynamical behaviors of the double-layered graphene are more complicated due to the van der Waals force between two layers. However, few studies about the thermal vibration of double-layered rectangle graphene considering quantum effects have been found. To this end, an equivalent plate model with quantum effects of a double-layered graphene is established. The van der Waals interaction between two adjacent layers can be simplified as Winkler type spring because the thermal amplitude of graphene is small when the temperature is very low. The nonlocal effects and initial stress of the graphene are also taken into consideration. The natural frequency and the root of mean squared (RMS) amplitude of the double-layered graphene with different initial stress in different temperatures are calculated. The in-phase vibration mode and anti-phase vibration mode can be obtained from thermal vibration of the double-layered graphene. The natural frequency of the in-phase vibration of the double-layered graphene is the same as the natural frequency of the single-layered graphene with the same size. The natural frequency of the anti-phase vibration is larger than the in-phase vibration frequency. The RMS amplitude obtained by the plate model considering quantum effects is smaller than that obtained by the plate model together with the law of energy equipartition. The different of these two models becomes more obvious at the lower temperature. The RMS amplitude of the anti-phase vibration is smaller than the in-phase vibration RMS amplitude. The peak values of the RMS amplitude of the anti-phase vibration in different models are very close because the natural frequency of anti-phase vibration increases slowly with the increasing of model. The natural frequency of the in-phase vibration will increase with increasing of the initial stress, while the RMS amplitude of the in-phase vibration will decrease. However, the changes of the natural frequency and the RMS amplitude of the anti-phase vibration are very small when the initial stress changes. The initial stress has more significant influence for the in-phase vibration than the anti-phase vibration. The effect of zero-point energy for the thermal vibration of the double-layered graphene is also investigated. If the zero-point energy is taken into consideration to the mean modal energy, the RMS amplitude obtained by the plate model considering quantum effects is larger than that obtained by the plate model together with the law of energy equipartition in a wide range. The difference is obvious when the temperature is very low. Besides, the RMS amplitude of the graphene is not zero at the temperature of 0 K.

double-layered graphene, thermal vibration, van der Waals' force, quantum effects, plate model, zero-point energy

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