论文

运动流体界面 Rayleigh-Taylor 不稳定性研究

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收稿日期: 2015-02-03; 接受日期: 2015-03-23; 网络出版日期: 2015-07-02

国家自然科学基金资助项目(批准号: 11129503)

摘要 本文从非相对论流体和相对论流体的基本方程组出发, 讨论了重力场下处于匀速运动的两种理想流体界面在随动参考系与实验室参考系下 Rayleigh-Taylor(RT)线性不稳定性的增长率. 计算表明, 非相对论流体 RT 不稳定增长率与参考系的选择无关. 相对论流体温度和压强对 RT 不稳定有抑制作用, 且相对论流体的速度也使该不稳定增长率下降.

关键词 Rayleigh-Taylor 不稳定, 运动界面, 相对论流体

PACS: 47.20.Ma, 47.75.+f, 52.35.Py

doi: 10.1360/SSPMA2015-00052

1 引言

流体力学不稳定性是一种常见的力学过程, 只要流场中有压力、密度、温度和速度的不均匀性, 就会产生流体力学不稳定性. 其他在天文、工程、物理等学科中都有着广泛的研究. 譬如在惯性约束聚变内爆中存在有 Rayleigh-Taylor (RT), Richtmyer-Meshkov (RM) 和 Kelvin-Helmholtz (KH) 三种流体不稳定性^[1-3]. 轻流体加速重流体或重力场中轻流体支撑重流体时, 流体界面是 RT 不稳定的, 该不稳定性最早由 Rayleigh^[4]从理论上给出, 由 Taylor^[5]在实验上予以证明. 在惯性约束聚变中, 当激光或 X 射线烧蚀靶丸球壳的外表面时, 在球壳周围形成一层稀薄等离子体, 烧蚀阵面把烧蚀层分成两区, 阵面里面是高密度区, 烧蚀阵面附近形成里面高、外面低的热压力梯度, 在烧蚀阵面产生了 RT 不稳定性, 称为激光烧蚀不稳

定^[6-9]. 在内爆后期的壳层连续减速阶段, 高温低密度热斑和低温高密度球壳之间的内界面又是 RT 不稳定的. 界面上的 RT 不稳定在内爆过程中会发展增大, 导致球壳破裂, 最终可能致使靶丸内爆失败^[10-12].

近年来, 在相对论强激光驱动离子加速的研究中, 人们利用高对比度圆偏振强激光与薄膜靶相互作用, 激光有质动力在靶前推动电子形成电子压缩层进入靶内, 电子压缩层与其后的离子层形成强静电场, 从而拉动并加速离子, 这被称为辐射压离子加速, 是目前的一个研究热点^[13-17]. 在这一加速过程中, 圆偏光电磁场推动靶相当于用轻流体加速重流体, 满足 RT 不稳定增长条件. 这一不稳定性的发展将破坏靶的结构, 使离子束的能散度变大, 准直性变差, 因此研究 RT 不稳定增长过程及如何抑制其增长对激光离子加速也有重要意义^[18-20].

引用格式: 刘晋陆, 盛政明. 运动流体界面 Rayleigh-Taylor 不稳定性研究. 中国科学: 物理学 力学 天文学, 2015, 45: 085201

Liu J L, Sheng Z M. Rayleigh-Taylor instability at the interface of moving fluids (in Chinese). Sci Sin-Phys Mech Astron, 2015, 45: 085201, doi: 10.1360/SSPMA2015-00052

很多文献在讨论激光与靶相互作用的 RT 不稳定增长时, 引入横向拉格朗日坐标标记平面靶面元位置, 将平面靶空间直角坐标表示为拉格朗日坐标的函数, 联立面元在有质动力作用下的动量方程, 得到面元的运动及一阶微扰后的不稳定性增长, 这种不稳定性称为类似 RT 不稳定性^[19].

本文提出一个初速度不为 0 的流体运动模型, 将流体初始静止时的参考系看作流体的初始随动坐标系, 则流体的初始速度即是这一随动坐标系相对于实验室坐标系的速度, 而在实验室坐标系中观察则相当于进行坐标变换. 从非相对论与相对论的流体力学基本方程出发, 采用轻流体在重力场中支撑重流体的模型研究随动系(初速为 0)与实验室系(初速不为 0)下的 RT 不稳定线性阶段色散关系, 所讨论的流体是无能量耗散的理想不可压缩流体.

2 非相对论流体的 RT 不稳定

非相对论流体是指流体速度很小 ($u \ll c$) 且流体粒子的内部微观运动速度也很小的流体^[20]. 在我们的模型中, 流体初速度及所受重力沿 Z 轴正向, 即 $\mathbf{u}_0 = u_0 \mathbf{e}_z$, $\mathbf{g} = -g \mathbf{e}_z$. 流体密度在界面两侧均匀, 仅在界面处存在沿 Z 轴正向的密度梯度. 在界面两侧的密度分别为 $\rho_{z<0}$ 和 $\rho_{z>0}$. 物理量下标为 0 表示初始值, 下标为 1 表示一阶扰动, $\rho_{z>0} + \rho_{z<0} = N^+$, $\rho_{z>0} - \rho_{z<0} = N^-$, 扰动量的形式均为

$$(u_x, u_y, u_z, P_1, \rho_1) = [u_x(z), u_y(z), u_z(z), P_1(z), \rho_1(z)] e^{(ik_x + ik_y - i\omega t)}.$$

图 1 (网络版彩图) Rayleigh-Taylor 不稳定性模型示意图, 其中流体速度, 加速度方向平行于密度梯度方向

Figure 1 (Color online) Schematic model of Rayleigh-Taylor instability, where the directions of fluid velocity and acceleration are parallel to the density gradient.

2.1 随动坐标系下的非相对论流体

随动坐标系下非相对论流体 Rayleigh-Taylor 不稳定色散关系为 $\omega^2 = -kg \frac{N^-}{N^+}$, 其中 $\rho_{z>0} + \rho_{z<0} = N^+$, $\rho_{z>0} - \rho_{z<0} = N^-$. 当 $\rho_{z>0} > \rho_{z<0}$ 时, $\omega^2 < 0$, 扰动将以指数形式增长, 发生流体力学的不稳定性^[20,21], 增长率为 $\gamma = \sqrt{kgN^- / N^+}$.

2.2 实验室坐标系下的非相对论流体

以上考虑的是静止流体, 但在实际研究中, 流体一般具有一定的初速度, 比如激光烧蚀不稳定性中的烧蚀对流速度. 微扰速度 $\mathbf{u}_1$ 是流体相对于随动系的速度, 而随动系相对于实验室参考系以 $\mathbf{u}_0$ 运动, 由伽利略变换可知, 微扰流体相对于实验室参考系的速度为 $\mathbf{u} = u_0 \mathbf{e}_z + \mathbf{u}_1$. 假定流体满足以下的流体方程:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = 0, \quad (1)$$

$$\rho \frac{\partial \mathbf{u}}{\partial t} + \rho (\mathbf{u} \cdot \nabla) \mathbf{u} = -\nabla P + \rho \mathbf{g}, \quad (2)$$

以及不可压缩条件: $\frac{d\rho}{dt} = 0$. 由伽利略速度合成公式, 微扰后的速度为 $\mathbf{u} = \mathbf{u}_0 + \mathbf{u}_1$, 线性化后得: $\frac{\partial \rho_1}{\partial t} +$

$$\nabla \cdot (\rho_0 \mathbf{u}_1 + \rho_1 \mathbf{u}_0) = 0, \quad \rho_0 \frac{\partial \mathbf{u}_1}{\partial t} + \rho_0 \mathbf{u}_0 \cdot \nabla \mathbf{u}_1 = -\nabla P_1 + \rho_1 \mathbf{g},$$

$\nabla \cdot \mathbf{u}_1 = 0$. 联立可得

$$\begin{aligned} & \frac{u_0^2}{gk^2} \frac{\partial^2}{\partial z^2} \left(\rho_0 \frac{\partial^2 u_{1z}}{\partial z^2} \right) - \frac{i u_0 \omega}{gk^2} \frac{\partial^2}{\partial z^2} \left(\rho_0 \frac{\partial u_{1z}}{\partial z} \right) \\ & - \frac{i \omega u_0}{gk^2} \frac{\partial}{\partial z} \left(\rho_0 \frac{\partial^2 u_{1z}}{\partial z^2} \right) - \left(\frac{\omega^2}{gk^2} + \frac{u_0^2}{g} \right) \frac{\partial}{\partial z} \left(\rho_0 \frac{\partial u_{1z}}{\partial z} \right) \\ & + \frac{i \omega u_0}{g} \frac{\partial}{\partial z} (\rho_0 u_{1z}) + \frac{i \omega u_0}{g} \left(\rho_0 \frac{\partial u_{1z}}{\partial z} \right) \\ & + \frac{\omega^2}{g} \rho_0 u_{1z} + u_{1z} \frac{\partial \rho_0}{\partial z} = 0, \end{aligned} \quad (3)$$

在界面以外区域, $\frac{\partial \rho_0}{\partial z} = 0$, 得通解为

$$u_{1z} = A e^{+kz} + B e^{-kz} + (C + Dz) e^{\frac{i\omega}{u_0} z}. \quad (4)$$

由 $|z| \rightarrow \pm\infty$ 时, $u_{1z} = 0$, 且在 $z = 0$ 处, u_{1z} 连续, 得 $u_{1z} = Ue^{+kz}, z \leq 0, u_{1z} = Ue^{-kz}, z \geq 0$. 将(3)式在界面处积分得

$$\begin{aligned} & \frac{u_0^2}{gk^2}(-Uk^3N^+) - \frac{i\omega_0\omega}{gk^2}(Uk^2N^-) - \frac{i\omega u_0}{gk^2}(Uk^2N^-) \\ & - \left(\frac{\omega^2}{gk^2} + \frac{u_0^2}{g} \right) (-UkN^+) + \frac{i\omega u_0}{g}(UN^-) \\ & + \frac{i\omega u_0}{g}(UN^-) + UN^- = 0, \end{aligned} \quad (5)$$

化简得: $\omega^2 = -kg \frac{N^-}{N^+}$, 即初速 $u_0 \neq 0$ 时的 Rayleigh-Taylor 不稳定性增长率, 与初速 $u_0 = 0$ 时的结论一样.

由非相对论运动流体的界面 Rayleigh-Taylor 不稳定性与初速度无关可知, 不稳定性的发展在伽利略变换下是不变的, 即 Rayleigh-Taylor 不稳定性增长率与所处的经典惯性系的选择无关.

3 相对论流体的 RT 不稳定

考虑相对论效应之所以必要, 不仅是由于流体的宏观运动速度可以很大, 而且也是由于流体中粒子的热速度很大, 此时相对论状态方程所描述的物质中压力与包括静止质量的内能密度是可以相比较的^[22]. 在这种情况下, 我们需要考虑相对论流体方程.

狭义相对论不考虑强引力, 时空几何近似为平直欧氏空间几何, 克里斯托弗符号为 0, 可以用普通导数代替协变导数. 取四维时空度规

$$\eta^{\alpha\beta} = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \text{ 在流体的随动坐标系中令流}$$

体密度为 $\rho_0(\mathbf{x}) = m_0 n_0$, 四维速度为 $U^\alpha(\mathbf{x}) = \frac{d\mathbf{x}^\alpha}{d\tau}$,

则在实验室坐标系中, 流体的相对论质量增加及流体体积在运动方向上的相对论压缩使得

$$\rho' = m'n' = \gamma m_0 \frac{N_{\text{total}}}{l^3 / \gamma} = \gamma^2 m_0 \frac{N_{\text{total}}}{l^3} = \gamma^2 \rho_0, \text{ 由任一惯性}$$

系中的四维速度形式 $U^\alpha = \gamma(c, \mathbf{u})$, ρ 可以看作一个二阶四维张量的 00 分量, 将这一张量写作 $T^{ij} = \rho_0(\mathbf{x})U^iU^j$ 即为流体的能量动量张量, 分量为:

$T^{00} = \gamma^2 \rho_0 c^2$ 表示粒子的能量密度, $T^{0i} = T^{i0} = \gamma^2 \rho_0 c U^i$ 是粒子的 i 方向的动量密度, $T^{ij} = \gamma^2 \rho_0 U^i U^j$ 当 $i = j$ 时表示 i 方向的压力, $i \neq j$ 表示黏性张力.

理想流体的 ρ 与 P 是标量, 则在随动参考系中的能量动量张量分量为: $T^{00} = \rho c^2$ 表示粒子的静止能量密度, $T^{0i} = T^{i0} = 0$, $T^{ij} = P \delta_{ij}$, P 表示静流体力压强, 即理想流体没有非各向同性张力, 其能量动量张量只有对角分量.

将能量动量张量变换到实验室参考系, 在该系中流体以速度 $\mathbf{u}$ 运动, 则由坐标变换 $x^\alpha = \Lambda^\alpha_\beta(\mathbf{v})x'^\beta$, 可得能量动量张量的变换为 $T^{\alpha\beta} = \Lambda^\alpha_\gamma(\mathbf{u})\Lambda^\beta_\delta(\mathbf{u})T'^{\gamma\delta}$,

$$\Lambda(\mathbf{u}) = \begin{pmatrix} \gamma & -\gamma \frac{u_j}{c} \\ -\gamma \frac{u_i}{c} & \delta_{ij} + \frac{(\gamma-1)u_i u_j}{u^2 c^2} \end{pmatrix}, \text{ 即 } T^{00} = \gamma^2 \left(\rho c^2 + \frac{P}{c^2} u^2 \right), T^{i0} = \gamma^2 \left(\rho + \frac{P}{c^2} \right) c u^i, T^{ij} = \gamma^2 \left(\frac{P}{c^2} + \rho \right) u^i u^j + P \delta_{ij}, \text{ 或写成张量形式: } T^{\mu\nu} = P \eta^{\mu\nu} + \left(\frac{P}{c^2} + \rho \right) U^\mu U^\nu.$$

相应的连续性方程、以及流体运动的能量和动量守恒方程为

$$\frac{\partial}{\partial t} \left[\rho \left(1 - \frac{u^2}{c^2} \right)^{-1/2} \right] + \nabla \cdot \left[\rho \mathbf{u} \left(1 - \frac{u^2}{c^2} \right)^{-1/2} \right] = 0, \quad (6)$$

$$\partial_\mu T^{\mu\nu} = \partial_\mu P + \partial_\mu \left[\left(\frac{P}{c^2} + \rho \right) U^\mu U^\nu \right] = 0. \quad (7)$$

方程(7)写成三维矢量方程和标量方程形式为

$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla) \mathbf{u} = - \frac{(1 - u^2/c^2)}{P/c^2 + \rho} \left(\nabla P + \frac{\mathbf{u}}{c^2} \frac{\partial P}{\partial t} \right), \quad (8)$$

$$\frac{\partial P}{\partial t} - \frac{\partial}{\partial t} \left(\frac{P/c^2 + \rho}{1 - u^2/c^2} \right) - \nabla \cdot \left(\frac{(P/c^2 + \rho) \mathbf{u}}{1 - u^2/c^2} \right) = 0. \quad (9)$$

由物态方程理论可知: $\rho - nm \approx (\Gamma - 1)^{-1} (P/c^2)$, 其中在非相对论情形 $\Gamma = 5/3$, 在极端相对论情形 $\Gamma = 4/3$, $\rho = \rho_0 + (3P/c^2)^{[22]}$. 定义流体的四维流密度矢量为 $j^\mu = \rho U^\mu$, 得到 Lorentz 标量 $j^2 = \eta_{\mu\nu} j^\mu j^\nu$

$= -\rho c^2$, 则质量守恒方程为: $\partial_\mu j^\mu = \frac{\partial}{\partial t}(\rho \gamma) + \nabla \cdot (\rho \mathbf{u} \gamma) = 0$.

3.1 随动系下的相对论流体

在流体的随动系中流体初始静止, 速度为微扰速度 $\mathbf{u} = \mathbf{u}_1 = (u_{1x}, u_{1y}, u_{1z}) \ll c$, 此时的流体速度是非相对论的: $(1 - \mathbf{u}^2)^{-1/2} = (1 - \mathbf{u}_1^2)^{-1/2} \approx 1$. 流体的四维速度为 $U^\mu = (c, u_{1x}, u_{1y}, u_{1z})$, 四维流密度为 $j^\mu = \rho_0 U^\mu$, ρ_0 为随动系中流体密度. 由此可以得到如下微扰方程

$$\nabla \cdot (\rho_0 \mathbf{u}_1) + \frac{\partial \rho_1}{\partial t} = 0, \quad (10)$$

$$\frac{\mathbf{u}_1}{c^2} \frac{\partial P_0}{\partial t} + \nabla P_1 = - \left(\frac{4P_0}{c^2} + \rho_0 \right) \frac{\partial \mathbf{u}_1}{\partial t} - g \left(\frac{4P_1}{c^2} + \rho_1 \right) \mathbf{e}_z, \quad (11)$$

$$\frac{d\rho}{dt} = \frac{\partial}{\partial t} \rho_1 + \mathbf{u}_1 \cdot \nabla \rho_0 = 0, \quad (12)$$

以上各式中 ρ 表示随动系中流体质量密度.

假定流体初始处于稳定状态 $\partial P / \partial t = 0$, 则不可压缩条件线性化为: $\frac{\partial \rho_1}{\partial t} + \mathbf{u}_1 \cdot \nabla \rho_0 = 0$, 与连续性方程

联立: $\nabla \cdot \mathbf{u}_1 = 0$, 同样由 $\frac{\partial \rho_0}{\partial x} = \frac{\partial \rho_0}{\partial y} = 0$, $\frac{\partial \rho_0}{\partial z} \neq 0$, 得线性化方程组为

$$(\partial \rho_1 / \partial t) + u_{1z} (\partial \rho_0 / \partial z) = 0, \quad (13)$$

$$\nabla P_1 = -(4P_0 / c^2 + \rho_0) (\partial \mathbf{u}_1 / \partial t) - g (4P_1 / c^2 + \rho_1) \mathbf{e}_z, \quad (14)$$

$$\nabla \cdot \mathbf{u}_1 = 0, \quad (15)$$

代入解的形式, 令 $4P_0 / c^2 + \rho_0 = \Sigma_0$, 得 $-i\omega \rho_1 +$

$$u_{1z} \left(\frac{\partial \rho_0}{\partial z} \right) = 0, \quad -i\omega \Sigma_0 u_{1x} = -ik_x P_1, \quad -i\omega \Sigma_0 u_{1y} = -ik_y P_1,$$

$$-i\omega \Sigma_0 u_{1z} = - \left(\frac{\partial P_1}{\partial z} \right) - g (4P_1 / c^2 + \rho_1), \quad ik_x u_{1x} + ik_y u_{1y} +$$

$$\left(\frac{\partial u_{1z}}{\partial z} \right) = 0. \text{ 令 } u_{1x} = 0, \text{ 则 } k_y = k, \text{ 得 } u_{1y} = \frac{i}{k} \frac{\partial u_{1z}}{\partial z}, P_1 =$$

$$i \frac{\Sigma_0}{k^2} \omega \frac{\partial u_{1z}}{\partial z}, \rho_1 = \frac{i\omega u_{1z} \Sigma_0}{g} - \frac{i\omega}{gk^2} \left[\frac{\partial \rho_0}{\partial z} \frac{\partial u_{1z}}{\partial z} + \Sigma_0 \frac{\partial^2 u_{1z}}{\partial z^2} \right].$$

关于 u_{1z} 的偏微分方程为: $\omega^2 \Sigma_0 u_{1z} = \frac{\omega^2 \Sigma_0}{k^2}$

$$\frac{\partial^2 u_{1z}}{\partial z^2} + \frac{\omega^2}{k^2} \frac{\partial \rho_0}{\partial z} \frac{\partial u_{1z}}{\partial z} - g \frac{\partial \rho_0}{\partial z} u_{1z}, \text{ 在非界面处 } \frac{\partial \rho_0}{\partial z} = 0,$$

得: $k^2 u_{1z} = \frac{\partial^2 u_{1z}}{\partial z^2}$, 通解形式为: $u_{1z} = Ae^{+kz} + Be^{-kz}$.

在远离界面的速度扰动为 0, 而在界面处速度扰动连续, 得: $u_{1z} = Ue^{+kz}, z \leq 0$; $u_{1z} = Ue^{-kz}, z \geq 0$. 令 $4P_0 / c^2 + \rho_0 = \Sigma^-$, $z \leq 0$; $4P_0 / c^2 + \rho_0 = \Sigma^+$, $z \geq 0$.

在界面处积分得色散关系: $\omega^2 = -\frac{gkN^-}{(\Sigma^+ + \Sigma^-)}$, 即不稳定性增长率为

$$\gamma = \sqrt{\frac{gkN^-}{(\Sigma^+ + \Sigma^-)}}, \quad (16)$$

此时色散关系与压强 P 有关, 热压力增大了流体惯性. 在经典情况下压强作用可以忽略, 就得到非相对论初速为 0 的增长率. 即在初速为 0 时, 与非相对论性流体相比, 相对论性流体的相对论热运动导致不稳定性增长率下降.

3.2 实验室坐标系下的相对论流体

实验室坐标系相对随动坐标系速度 $-\mathbf{u}_0 = -u_0 \mathbf{e}_z \neq 0$, 由速度的 Lorentz 变换得流体在实验室坐标系中的速度为 $u_x = u_{1x} \sqrt{1 - (u_0^2 / c^2)} = u_{1x} / \gamma_0$,

$$u_y = u_{1y} \sqrt{1 - \frac{u_0^2}{c^2}} = \frac{u_{1y}}{\gamma_0}, \quad u_z \approx (u_{1z} + u_0) \left(1 - \frac{u_{1z} u_0}{c^2} \right) = \frac{u_{1z}}{\gamma_0^2} +$$

$$u_0, \quad \gamma = \left(1 - \frac{\mathbf{u}^2}{c^2} \right)^{-1/2} = \gamma_0 \left(1 + \frac{u_0 u_{1z}}{c^2} \right), \text{ 流体的四维速度}$$

为 $U^\mu = \gamma \left(c, \frac{u_{1x}}{\gamma_0}, \frac{u_{1y}}{\gamma_0}, \frac{u_{1z}}{\gamma_0^2} + u_0 \right)$, 四维流密度矢量为

$j^\mu = \rho_0 U^\mu$ 则在实验室参考系中连续性方程为

$$\frac{\partial(\rho\gamma)}{\partial t} + \nabla \cdot (\rho \mathbf{u} \gamma) = 0, \quad (17)$$

动量方程为

$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla) \mathbf{u} + g \mathbf{e}_z = - \frac{(1 - \mathbf{u}^2 / c^2)}{4P / c^2 + \rho_0} \left(\nabla P + \frac{\mathbf{u}}{c^2} \frac{\partial P}{\partial t} \right). \quad (18)$$

流体的不可压缩性是指流体的体积不可压缩, 即单位流体体积内的流体粒子数目不变, 这种不可压缩性是指粒子数密度不可压缩, 因此虽然在实验室坐标系下, 随动系中的粒子质量密度由于相对论质量增加及相对论体积压缩应写为 $\rho\gamma^2$, 但这里不可压缩条件应写为: $\frac{d(\rho\gamma)}{dt} = 0$, 即只考虑相对论体积压缩效应, 以上各式中 ρ 表示随动系中流体质量密度.

不可压缩条件为: $\frac{\partial(\rho\gamma)}{\partial t} + \mathbf{u} \cdot \nabla(\rho\gamma) = 0$, 不可压缩条件与连续性方程联立得连续性方程: $\rho\gamma \nabla \cdot \mathbf{u} = 0$, 假定流体初始处于平衡状态 $\partial P_0 / \partial t = 0$, 且

$\nabla P_0 = -\left(\frac{4P_0}{c^2} + \rho_0\right)g\mathbf{e}_z = -\Sigma_0 g\mathbf{e}_z$, 可得连续性方程:

$$\frac{\partial u_{1x}}{\partial x} + \frac{\partial u_{1y}}{\partial y} + \frac{1}{\gamma_0} \frac{\partial u_{1z}}{\partial z} = 0. \quad (19)$$

不可压缩条件为

$$\begin{aligned} & \left(\frac{\partial \rho_1}{\partial t} + u_0 \frac{\partial \rho_1}{\partial z} \right) + \frac{\rho_0 u_0}{c^2} \left(\frac{\partial u_{1z}}{\partial t} + u_0 \frac{\partial u_{1z}}{\partial z} \right) \\ & + \frac{u_{1z}}{\gamma_0^2} \frac{\partial \rho_0}{\partial z} = 0. \end{aligned} \quad (20)$$

令 $u_{1x} = 0$, 得动量方程分量形式:

$$\begin{aligned} & \left(-i\omega \rho_1 + u_0 \frac{\partial \rho_1}{\partial z} \right) + \frac{\rho_0 u_0}{c^2} \left(-i\omega u_{1z} + u_0 \frac{\partial u_{1z}}{\partial z} \right) \\ & + \frac{u_{1z}}{\gamma_0^2} \frac{\partial \rho_0}{\partial z} = 0, \end{aligned} \quad (21)$$

$$iku_{1y} + \frac{1}{\gamma_0} \frac{\partial u_{1z}}{\partial z} = 0, \quad (22)$$

$$\left(-i\omega u_{1y} + \frac{u_0}{\gamma_0} \frac{\partial u_{1y}}{\partial z} \right) \Sigma_0 = -\frac{1}{\gamma_0^2} ikP_1, \quad (23)$$

$$\begin{aligned} & \left(-i\omega u_{1z} + \frac{u_0}{\gamma_0^2} \frac{\partial u_{1z}}{\partial z} + \frac{2u_0 u_{1z}}{\gamma_0^2 c^2} g \right) \Sigma_0 + g\rho_1 \\ & = -\frac{1}{\gamma_0^2} \frac{\partial P_1}{\partial z} + \left(\frac{i\omega u_0}{\gamma_0^2 c^2} - \frac{4g}{c^2} \right) P_1. \end{aligned} \quad (24)$$

将 u_{1y} , P_1 , ρ_1 代入(20), 其中 $\frac{\partial \Sigma_0}{\partial z} = -\frac{4g}{c^2} \Sigma_0 + \frac{\partial \rho_0}{\partial z}$, 除

界面外 $\frac{\partial \rho_0}{\partial z} = 0$, 可得关于 u_{1z} 的偏微分方程:

$$\begin{aligned} & \left(i\omega - \frac{2u_0 g}{(\gamma_0 c)^2} \right) \left(-\frac{i\omega}{g} u_{1z} + \frac{u_0}{g} \frac{\partial u_{1z}}{\partial z} - \frac{4u_0}{c^2} u_{1z} \right) \Sigma_0 \\ & + \left(-\frac{u_0}{\gamma_0^2} + \frac{i\omega 4g}{(kc)^2 \gamma_0} - \frac{\omega^2 u_0}{(kc)^2 \gamma_0} - \frac{i\omega 4g \gamma_0}{(kc)^2} \right) \left(-\frac{i\omega}{g} \frac{\partial u_{1z}}{\partial z} + \frac{u_0}{g} \frac{\partial^2 u_{1z}}{\partial z^2} - \frac{4u_0}{c^2} \frac{\partial u_{1z}}{\partial z} \right) \Sigma_0 \\ & + \left(-\frac{i\omega}{k^2 \gamma_0} - \frac{4gu_0}{(kc\gamma_0)^2} - \frac{i\omega u_0^2}{(kc\gamma_0)^2} + \frac{4gu_0}{(kc)^2} \right) \left(-\frac{i\omega}{g} \frac{\partial^2 u_{1z}}{\partial z^2} + \frac{u_0}{g} \frac{\partial^3 u_{1z}}{\partial z^3} - \frac{4u_0}{c^2} \frac{\partial^2 u_{1z}}{\partial z^2} \right) \Sigma_0 \\ & + \frac{u_0}{(k\gamma_0)^2} \left(-\frac{i\omega}{g} \frac{\partial^3 u_{1z}}{\partial z^3} + \frac{u_0}{g} \frac{\partial^4 u_{1z}}{\partial z^4} - \frac{4u_0}{c^2} \frac{\partial^3 u_{1z}}{\partial z^3} \right) \Sigma_0 + \frac{\rho_0 u_0^2}{c^2} \frac{\partial u_{1z}}{\partial z} - \frac{i\omega \rho_0 u_0 u_{1z}}{c^2} = 0. \end{aligned} \quad (25)$$

方程(25)的通解仍为: $u_{1z} = Ae^{+kz} + Be^{-kz}$. 在远离界面的速度扰动为 0, 而在界面处速度扰动连续, 则: $u_{1z} = Ue^{+kz}, z \leq 0; u_{1z} = Ue^{-kz}, z \geq 0$. 在界面处的方程为

$$\begin{aligned} & \left(i\omega - \frac{u_0 g}{(\gamma_0 c)^2} \right) \left(-\frac{i\omega}{g} \Sigma_0 u_{1z} + \frac{u_0}{g} \Sigma_0 \frac{\partial u_{1z}}{\partial z} - \frac{4u_0}{c^2} \Sigma_0 u_{1z} + \frac{u_0}{g} \frac{\partial \rho_0}{\partial z} u_{1z} \right) \\ & + \left(-\frac{u_0}{\gamma_0^2} + \frac{i\omega 4g}{(kc)^2 \gamma_0} - \frac{\omega^2 u_0}{(kc)^2 \gamma_0} - \frac{i\omega 4g \gamma_0}{(kc)^2} \right) \left(-\frac{i\omega}{g} \Sigma_0 \frac{\partial u_{1z}}{\partial z} + \frac{u_0}{g} \Sigma_0 \frac{\partial^2 u_{1z}}{\partial z^2} - \frac{4u_0}{c^2} \Sigma_0 \frac{\partial u_{1z}}{\partial z} + \frac{u_0}{g} \frac{\partial \rho_0}{\partial z} \frac{\partial u_{1z}}{\partial z} \right) \\ & + \left(-\frac{i\omega}{k^2 \gamma_0} - \frac{4gu_0}{(kc\gamma_0)^2} - \frac{i\omega u_0^2}{(kc\gamma_0)^2} + \frac{4gu_0}{(kc)^2} \right) \left(-\frac{i\omega}{g} \Sigma_0 \frac{\partial^2 u_{1z}}{\partial z^2} + \frac{u_0}{g} \Sigma_0 \frac{\partial^3 u_{1z}}{\partial z^3} - \frac{4u_0}{c^2} \Sigma_0 \frac{\partial^2 u_{1z}}{\partial z^2} + \frac{u_0}{g} \frac{\partial \rho_0}{\partial z} \frac{\partial^2 u_{1z}}{\partial z^2} \right) \\ & + \frac{u_0}{(k\gamma_0)^2} \left(-\frac{i\omega}{g} \Sigma_0 \frac{\partial^3 u_{1z}}{\partial z^3} + \frac{u_0}{g} \Sigma_0 \frac{\partial^4 u_{1z}}{\partial z^4} - \frac{4u_0}{c^2} \Sigma_0 \frac{\partial^3 u_{1z}}{\partial z^3} + \frac{u_0}{g} \frac{\partial \rho_0}{\partial z} \frac{\partial^3 u_{1z}}{\partial z^3} \right) - \frac{\omega^2}{gk^2 \gamma_0} \frac{\partial \rho_0}{\partial z} \frac{\partial u_{1z}}{\partial z} \\ & + \frac{u_0^2}{(k\gamma_0)^2} \frac{\partial}{\partial z} \left(\frac{\partial \rho_0}{\partial z} \frac{\partial^2 u_{1z}}{\partial z^2} \right) - \frac{i\omega u_0}{gk^2 \gamma_0^2} \frac{\partial \rho_0}{\partial z} \frac{\partial^2 u_{1z}}{\partial z^2} - \frac{i\omega u_0}{k^2 \gamma_0} \frac{\partial}{\partial z} \left(\frac{\partial \rho_0}{\partial z} \frac{\partial u_{1z}}{\partial z} \right) + \frac{\rho_0 u_0^2}{c^2} \frac{\partial u_{1z}}{\partial z} - \frac{i\omega \rho_0 u_0 u_{1z}}{c^2} + \frac{u_{1z}}{(\gamma_0 c)^2} \frac{\partial \rho_0}{\partial z} = 0. \end{aligned} \quad (26)$$

对式(26)在界面处积分得

$$\omega^2(\Sigma^+ + \Sigma^-) + gkN^-[1 - u_0^2/(\gamma_0 c)^2] = 0. \quad (27)$$

解方程(27)得到实验室参考系下 RT 不稳定性增长率为

$$\gamma = \sqrt{\frac{gkN^-[1 - u_0^2/(\gamma_0 c)^2]}{(\Sigma^+ + \Sigma^-)}}. \quad (28)$$

当 $u_0 = 0$ 时, 方程(28)即回到方程(16). 当 $u_0 \ll c$ 时, $1 - u_0^2/(\gamma_0 c)^2 \approx 1 - u_0^2/c^2$ 表明相对论效应增大了流体惯性, 与非相对论性流体相比, 相对论运动导致不稳定增长率下降. 这种情况下, 这个相对论修正系数与相对论变换对应的时间变换是一致的. 需要说明的是, 方程(28)中的 $1 - u_0^2/(\gamma_0 c)^2$ 因子在 $(u_0/c)^2 \geq 1/2$ 后值开始变大, 在物理上是不合理的. 这是因为在解(25)式时, 为得到解析解, 做了近似, 在计算过程中, 忽略了关于 u_{1z} 的高次微分.

在惯性约束聚变内爆过程中^[7], 其加速度可以达到 $g = 2 \times 10^{15} \text{ m/s}^2$, 流体的运动速度可以达到 400 km/s. 但这仍远小于光速, 所以其线性增长率减小效果不明显. 在人们探讨的强激光与固体薄膜靶作用中^[14-15], 强激光对薄膜靶产生的加速度可以近似表达为 $g = 2a_0^2(m_e n_e c^2 / m_i n_i d_0)$, 其中 a_0 是归一化激光

场矢势振幅, m_e 和 m_i 分别是电子和离子质量, n_e 和 n_i 分别是激光波长对应的等离子体临界密度和固体靶初始密度, c 是真空中光速, d_0 是薄膜靶初始靶厚度. 如果取 $a_0 = 100$ (对应光强约为 10^{22} W/cm^2), $m_i/m_e = 1836$, $n_i = 100n_e$, $d_0 = 10 \text{ nm}$, 则 $g = 9.8 \times 10^{23} \text{ m/s}^2$. 经过几十个光周期的加速时间^[14], 质子动能可以达到 1 GeV. 对应束流的相对论因子可以达到 $\gamma_0 \sim 2$, RT 不稳定增长率将明显减小.

4 结论

本文从流体力学方程出发推导了运动界面中的 RT 不稳定增长率. 计算表明, 非相对论流体 RT 不稳定增长率与参考系的选择无关. 采用相对论流体方程, 给出了普遍性的 RT 不稳定增长率公式, 显示相对论温度和压强对 RT 不稳定增长有抑制作用, 且宏观流体的运动将使该不稳定增长变慢. 这从物理上可以理解为, 当相对论流体的微观粒子热运动速度很大, 宏观上表现为压强对流体惯性的贡献不可忽略. 当流体的初速度不为零时, 扰动在流体中的增长将进一步由于实验室坐标系下等效质量的相对论增加而受到抑制, 这部分的修正与对应不同坐标系中时间变换相对应.

致谢 本文初稿成文于 2010 年冬, 期间曾得到王立峰博士的仔细阅读和指正, 在此表示衷心感谢.

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Rayleigh-Taylor instability at the interface of moving fluids

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Starting from the nonrelativistic fluid and relativistic fluid models, we study the linear growth of the Rayleigh-Taylor instability (RTI) at the interface of two ideal fluids moving with certain speed under a gravitational field, where the fluid velocity is along the gravitational field and is perpendicular to interface. Calculation shows that, the RTI growth rate for nonrelativistic fluids is independent of its reference frame or the fluid velocity. A general formula of the RTI growth rate is obtained for relativistic fluids. It shows that both the relativistic thermal pressure and macroscopic fluid velocity tend to suppress the RTI growth. Physically, when the thermal velocities of microparticles of the relativistic fluids are considerable as compared to the light speed in vacuum, the contribution of the thermal pressure to the fluid inertia cannot be ignored. Also when the fluid velocities are not negligible as compared to the light speed, the effect of relativistic-mass-increase leads to the decreased RTI growth rate. The latter is basically consistent with the time transformation effect in different reference frames.

Rayleigh-Taylor instability, moving interface, relativistic fluid

PACS: 47.20.Ma, 47.75.+f, 52.35.Py

doi: 10.1360/SSPMA2015-00052