第一类华罗庚域上 u -Bloch 空间到 v -Bloch 空间复合算子的有界性和紧性

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收稿日期: 2015-03-30; 接受日期: 2015-07-23; * 通信作者

国家自然科学基金 (批准号: 11171285) 资助项目

摘要 本文在第一类华罗庚域上定义了一个加权 Bloch 空间, 讨论了第一类华罗庚域上 u -Bloch 空间到 v -Bloch 空间的复合算子的有界性和紧性, 分别得到了复合算子是有界算子和紧算子的充分条件和必要条件.

关键词 华罗庚域 加权 Bloch 空间 复合算子的有界性和紧性

MSC (2010) 主题分类 32A18, 47B33

1 引言

设 $\Omega \subset \mathbb{C}^n$ 是有界域, Ω 上所有全纯函数组成的集合记为 $\text{Hol}(\Omega)$. ϕ 为 Ω 上的全纯自映照, μ 为 Ω 上的全纯函数, 则定义加权复合算子 μC_ϕ 为 $\mu C_\phi f = \mu f \circ \phi$, $f \in \text{Hol}(\Omega)$ (参见文献 [1]). 当 $\mu \equiv 1$ 时, 加权复合算子就是通常的复合算子 [2]. 设 $\mathbb{B}$ 为 $\mathbb{C}^n$ 中的单位超球, 对于 $f \in \text{Hol}(\mathbb{B})$, 定义

$$\|f\|_{p,1} = |f(0)| + \sup_{z \in \mathbb{B}} (1 - |z|^2)^p Q_f(z) < \infty, \quad (1.1)$$

$$\|f\|_{p+1,2} = |f(0)| + \sup_{z \in \mathbb{B}} (1 - |z|^2)^{p+1} |\nabla f(z)| < \infty, \quad (1.2)$$

这里

$$\nabla f(z) = \left(\frac{\partial f}{\partial z_1}(z), \frac{\partial f}{\partial z_2}(z), \dots, \frac{\partial f}{\partial z_n}(z) \right), \quad Q_f(z) = \sup \left\{ \frac{|\nabla f(z)u|}{H_z^{1/2}(u,u)} : u \in \mathbb{C}^n - \{0\} \right\},$$

$H_z(u, u)$ 为 $\mathbb{B}$ 上的 Bergman 度量. 可以证明 $\|f\|_{p,1}$ 和 $\|f\|_{p+1,2}$ 是等价的 [3]. 称 $\text{Hol}(\mathbb{B})$ 中满足 (1.1) 或者 (1.2) 的函数全体所组成的函数空间为加权 Bloch 空间, 或进一步称为 p -Bloch 空间. 当 $p = 0$ 时, 加权 Bloch 空间就是通常的 Bloch 空间. (加权) Bloch 空间概念可以推广到有界齐性域上 [4-9].

各种有界域上 (加权) Bloch 空间之间或者 (加权) Bloch 空间和其他函数空间之间的 (加权) 复合算子的有界性和紧性以及相关问题, 近年来一直受到数学工作者的关注和研究. 例如, 文献 [10, 11] 研

英文引用格式: Su J B, Li H, Wang H. Boundedness and compactness of the composition operators from u -Bloch space to v -Bloch space on the first Hua domains (in Chinese). Sci Sin Math, 2015, 45: 1909-1918, doi: 10.1360/N012015-00146

究单位圆盘上、文献 [12–16] 研究单位超球上、文献 [17–19] 研究多圆柱上、文献 [4–9] 研究有界对称域上 (加权) Bloch 空间之间 (加权) 复合算子的有界性和紧性.

自 1998 年以来, 殷慰萍和 Roos 创建了 Cartan-Hartogs 域. 此后, 殷慰萍进一步依次创建了 Cartan-Egg 域、华罗庚域、广义华罗庚域和华罗庚结构. 其后一类域都是前一类域的推广, 现在这五类域统称为华罗庚域. 华罗庚域的创建, 统一了多复变中的对称典型域和蛋型域的研究. 除了超球等特殊情形, 一般来讲, 华罗庚域不是可递域 [20]. 第一类华罗庚域定义如下:

$$\mathrm{HE}_I(N_1, \dots, N_r; m, n; p_1, \dots, p_r) = \left\{ \xi_j \in \mathbb{C}^{N_j}, Z \in \mathfrak{R}_I(m, n) : \sum_{j=1}^r |\xi_j|^{2p_j} < \det(I - Z\bar{Z}^T) \right\}, \quad (1.3)$$

其中 $\mathfrak{R}_I(m, n) = \{Z \in \mathbb{C}^{mn} : I - Z\bar{Z}^T > 0\}$. 在不引起混淆的情形下, 我们将记之为 HE_I .

定义 1.1 设 $u \in \mathbb{R}$, 若 HE_I 上的全纯函数 $f \in \mathrm{Hol}(\mathrm{HE}_I)$ 满足

$$\|f\|_{\beta^u} = \sup_{(Z, \xi) \in \mathrm{HE}_I} \left[\det(I - Z\bar{Z}^T) - \sum_{j=1}^r |\xi_j|^{2p_j} \right]^u |\nabla f(Z, \xi)| < +\infty, \quad (1.4)$$

其中

$$\begin{aligned} \nabla f(Z, \xi) &= \left(\frac{\partial f(Z, \xi)}{\partial z_{11}}, \dots, \frac{\partial f(Z, \xi)}{\partial z_{mn}}, \frac{\partial f(Z, \xi)}{\partial \xi_{11}}, \dots, \frac{\partial f(Z, \xi)}{\partial \xi_{rN_r}} \right), \\ Z &= (z_{ij})_{m \times n}, \quad \xi = (\xi_1, \dots, \xi_r), \quad \xi_j = (\xi_{j1}, \dots, \xi_{jN_j}), \quad j = 1, \dots, r, \end{aligned}$$

则称 f 属于 HE_I 上的 u -Bloch 空间, 记为 $f \in \beta^u(\mathrm{HE}_I)$,

$$\|f\|_u = |f(0, 0)| + \|f\|_{\beta^u}. \quad (1.5)$$

类似于超球上的情形, 可以证明 $\|f\|_u$ 是一个范数, 关于此范数, $\beta^u(\mathrm{HE}_I)$ 是一个 Banach 空间.

显然, 当 $u < 0$ 时, $f \in \beta^u(\mathrm{HE}_I)$ 是一个常函数, 因此, 我们不妨假设 $u \geq 0$. 本文主要讨论第一类华罗庚域上 u -Bloch 空间到 v -Bloch 空间的复合算子的有界性和紧性.

本文剩余部分结构如下: 第 2 节给出需要的一些引理; 第 3 节讨论第一类华罗庚域上 u -Bloch 空间到 v -Bloch 空间的复合算子的有界性; 第 4 节讨论第一类华罗庚域上 u -Bloch 空间到 v -Bloch 空间的复合算子的紧性.

2 引理

为书写方便, 我们在以下引理以及定理中取 $N_j = 1$, 即 $\xi_j \in \mathbb{C}$ ($j = 1, 2, \dots, r$). 我们的结论对一般的情形也成立. 设 $\phi = (\psi_{11}, \psi_{12}, \dots, \psi_{mn}, \varphi_1, \dots, \varphi_r)$ 为 HE_I 上的全纯自映照, $\phi'(Z, \xi)$ 是 ϕ 的 Jacobi 矩阵, $|\phi'(Z, \xi)|$ 表示矩阵的一般模, 即

$$\begin{aligned} |\phi'(Z, \xi)|^2 &= \sum_{\substack{1 \leq \alpha \leq m \\ 1 \leq \beta \leq n}} \sum_{\substack{1 \leq k \leq m \\ 1 \leq l \leq n}} \left| \frac{\partial \psi_{kl}(Z, \xi)}{\partial z_{\alpha\beta}} \right|^2 + \sum_{1 \leq \gamma \leq r} \sum_{\substack{1 \leq k \leq m \\ 1 \leq l \leq n}} \left| \frac{\partial \psi_{kl}(Z, \xi)}{\partial \xi_\gamma} \right|^2 \\ &+ \sum_{\substack{1 \leq \alpha \leq m \\ 1 \leq \beta \leq n}} \sum_{1 \leq \tau \leq r} \left| \frac{\partial \varphi_\tau(Z, \xi)}{\partial z_{\alpha\beta}} \right|^2 + \sum_{1 \leq \gamma \leq r} \sum_{1 \leq \tau \leq r} \left| \frac{\partial \varphi_\tau(Z, \xi)}{\partial \xi_\gamma} \right|^2. \end{aligned}$$

定义

$$H(Z, \xi; W, \eta; |\phi'(Z, \xi)|) = \frac{[\det(I - Z\bar{Z}^T) - \sum_{j=1}^r |\xi_j|^{2p_j}]^v}{[\det(I - W\bar{W}^T) - \sum_{j=1}^r |\eta_j|^{2p_j}]^u} |\phi'(Z, \xi)|. \quad (2.1)$$

引理 2.1 如果 $H(Z, \xi; W, \eta; |\phi'(Z, \xi)|) = O(1)$, $(Z, \xi) \in \text{HE}_I$, $(W, \eta) = \phi(Z, \xi) \rightarrow \partial\text{HE}_I$ 成立, 则存在常数 C , 使得

$$H(Z, \xi; W, \eta; |\phi'(Z, \xi)|) \leq C, \quad (Z, \xi) \in \text{HE}_I, \quad (W, \eta) = \phi(Z, \xi). \quad (2.2)$$

证明 由条件知, 存在一个常数 $\sigma > 0$, 当 $\text{dist}((W, \eta), \partial\text{HE}_I) < \sigma$ 时, 存在常数 $C_1 > 0$, 使得 $H(Z, \xi; W, \eta; |\phi'(Z, \xi)|) \leq C_1$, $(Z, \xi) \in \text{HE}_I$.

令 $E_\sigma = \{(W, \eta) \in \text{HE}_I : \text{dist}((W, \eta), \partial\text{HE}_I) \geq \sigma\}$. 易知 E_σ 为 HE_I 的一个紧子集, 因此存在一个常数 $M \in (0, 1)$, 使得 $M \leq \det(I - W\bar{W}^T) - \sum_{j=1}^r |\eta_j|^{2p_j} \leq 1$. 因而存在常数 $C_2 > 0$, 使得 $H(Z, \xi; W, \eta; |\phi'(Z, \xi)|) \leq C_2$.

令 $C = \max\{C_1, C_2\}$, 则对一切 $(Z, \xi) \in \text{HE}_I$, $(W, \eta) = \phi(Z, \xi)$, 有 $H(Z, \xi; W, \eta; |\phi'(Z, \xi)|) \leq C$. $\square$

引理 2.2 设 $u \geq 0$, K 为 HE_I 的一个紧子集, 则存在一个常数 $C(K) > 0$, 使得对任意的 $(Z, \xi) \in K$, 有

$$\int_0^1 \frac{|(Z, \xi)|}{[\det(I - t^2 Z\bar{Z}^T) - \sum_{j=1}^r |t\xi_j|^{2p_j}]^u} dt < C. \quad (2.3)$$

证明 当 $u = 0$ 时, 有

$$\int_0^1 \frac{|(Z, \xi)|}{[\det(I - Z\bar{Z}^T) - \sum_{j=1}^r |\xi_j|^{2p_j}]^u} dt = \int_0^1 |(Z, \xi)| dt = |(Z, \xi)| \leq \sqrt{m+r}.$$

当 $u > 0$ 时, 令 $E_\sigma = \{(Z, \xi) \in \text{HE}_I : \det(I - Z\bar{Z}^T) - \sum_{j=1}^r |\xi_j|^{2p_j} \geq \sigma\}$, $\sigma \in (0, 1)$, 则对于紧集 $K \subset \text{HE}_I$, 存在一个常数 $\sigma_K \in (0, 1)$, 使得 $K \subset E_{\sigma_K}$. 对任意 $(Z, \xi) \in K$, $t \in [0, 1]$, 我们有

$$\sum_{j=1}^r |t\xi_j|^{2p_j} \leq \sum_{j=1}^r |\xi_j|^{2p_j} \leq \det(I - Z\bar{Z}^T) \leq \det(I - t^2 Z\bar{Z}^T).$$

因此,

$$0 < \frac{1}{\det(I - t^2 Z\bar{Z}^T) - \sum_{j=1}^r |t\xi_j|^{2p_j}} \leq \frac{1}{\det(I - Z\bar{Z}^T) - \sum_{j=1}^r |\xi_j|^{2p_j}} \leq \frac{1}{\sigma_K}.$$

进一步可得

$$\begin{aligned} \int_0^1 \frac{|(Z, \xi)|}{[\det(I - t^2 Z\bar{Z}^T) - \sum_{j=1}^r |t\xi_j|^{2p_j}]^u} dt &\leq \int_0^1 \frac{|(Z, \xi)|}{[\det(I - Z\bar{Z}^T) - \sum_{j=1}^r |\xi_j|^{2p_j}]^u} dt \\ &\leq \frac{1}{\sigma_K^u} \int_0^1 |(Z, \xi)| dt < \frac{1}{\sigma_K^u} \sqrt{m+r} =: C. \end{aligned}$$

引理 2.2 证毕. $\square$

引理 2.3 设 K 为 HE_I 的一个紧子集, 则对任意 $(Z, \xi) \in K$, $f \in \beta^u(\text{HE}_I)$, 存在常数 $C > 0$, 使得 $|f(Z, \xi)| \leq C\|f\|_u$.

证明 因为

$$\frac{\partial f(tZ, t\xi)}{\partial t} = \frac{\partial f(tZ, t\xi)}{\partial v_{11}} z_{11} + \frac{\partial f(tZ, t\xi)}{\partial v_{12}} z_{12} + \cdots + \frac{\partial f(tZ, t\xi)}{\partial v_{mn}} z_{mn} + \frac{\partial f(tZ, t\xi)}{\partial v_{mn+1}} \xi_1 + \cdots + \frac{\partial f(tZ, t\xi)}{\partial v_{mn+r}} \xi_r$$

$$= \langle \nabla f(tZ, t\xi), \overline{(Z, \xi)} \rangle,$$

其中 $(Z, \xi) = (z_{11}, z_{12}, \dots, z_{mn}, \xi_1, \dots, \xi_r)$, 由引理 2.2 可知, 存在一个常数 $C > 0$, 使得

$$\begin{aligned} |f(Z, \xi)| &= \left| f(0, 0) + \int_0^1 \langle \nabla f(tZ, t\xi), \overline{(Z, \xi)} \rangle dt \right| \leq |f(0, 0)| + \int_0^1 |\nabla f(tZ, t\xi)| |\overline{(Z, \xi)}| dt \\ &\leq |f(0, 0)| + \|f\|_{\beta^u} \int_0^1 \frac{|(Z, \xi)|}{[\det(I - t^2 Z \bar{Z}^T) - \sum_{j=1}^r |t\xi_j|^{2p_j}]^u} dt = C\|f\|_u. \end{aligned}$$

引理 2.3 证毕. □

引理 2.4 设 $(Z, \xi), (W, \eta) \in \text{HE}_I$. 记 $\|\xi\|^2 = \sum_{j=1}^r |\xi_j|^{2p_j}$, $\|\eta\|^2 = \sum_{j=1}^r |\eta_j|^{2p_j}$, 则

$$[\det(I - Z \bar{Z}^T) - \|\xi\|^2][\det(I - W \bar{W}^T) - \|\eta\|^2] \leq [|\det(I - Z \bar{W}^T)| - \|\xi\| \|\eta\|]^2.$$

证明 当 a, b, c 和 d 都是非负实数, 且 $b \leq a, d \leq c$ 时, 容易验证 $(a^2 - b^2)(c^2 - d^2) \leq (ac - bd)^2$ 成立. 结合此不等式和著名的华罗庚不等式, 有

$$\begin{aligned} [\det(I - Z \bar{Z}^T) - \|\xi\|^2][\det(I - W \bar{W}^T) - \|\eta\|^2] &\leq [\det(I - Z \bar{Z}^T)^{\frac{1}{2}} \det(I - W \bar{W}^T)^{\frac{1}{2}} - \|\xi\| \|\eta\|]^2 \\ &\leq [|\det(I - Z \bar{W}^T)| - \|\xi\| \|\eta\|]^2. \end{aligned}$$

引理 2.4 证毕. □

可以利用文献 [21] 中类似方法证明下面的引理.

引理 2.5 对于从 $\beta^u(\text{HE}_I)$ 到 $\beta^v(\text{HE}_I)$ 的复合算子 C_ϕ , 下列条件等价:

- (i) C_ϕ 是从 $\beta^u(\text{HE}_I)$ 到 $\beta^v(\text{HE}_I)$ 的紧算子;
- (ii) 对于 $\beta^u(\text{HE}_I)$ 中, 在 HE_I 的紧子集上一致收敛于 0 的任一有界函数列 $\{f_t\}$, 都有 $\|C_\phi f_t\|_{\beta^v} \rightarrow 0, t \rightarrow \infty$.

证明 (i) $\rightarrow$ (ii). 若 $C_\phi : \beta^u(\text{HE}_I) \rightarrow \beta^v(\text{HE}_I)$ 是紧的. 设 $\{f_t\}$ 是 $\beta^u(\text{HE}_I)$ 中的有界函数列, 并且在 HE_I 的紧子集上一致收敛于 0.

假设 $\|C_\phi f_t\|_{\beta^v} \not\rightarrow 0, t \rightarrow \infty$, 那么存在一个子列 $\{f_{t_j}\} \subset \{f_t\}$, 使得 $\inf_j \|C_\phi f_{t_j}\|_{\beta^v} > 0$. 由于 C_ϕ 是紧的, 所以存在 $\{f_{t_j}\}$ 的一个子列, 不妨仍记为 $\{f_{t_j}\}$, 使得

$$\lim_{j \rightarrow \infty} \|g - C_\phi f_{t_j}\|_{\beta^v} = 0, \quad g \in \beta^v(\text{HE}_I)$$

成立. 设 K 是 HE_I 中的任意紧子集, 由引理 2.3 知, 存在一个常数 $C(K) > 0$, 使得

$$|(g - C_\phi f_{t_j})(Z, \xi)| \leq C(K) \|g - C_\phi f_{t_j}\|_{\beta^v} \rightarrow 0, \quad j \rightarrow \infty, \quad (Z, \xi) \in K.$$

因此, $\{g - C_\phi f_{t_j}\}$ 在紧子集 K 上一致收敛于 0, 则对 $\forall \varepsilon > 0$, 存在 $J_1 > 0$, 当 $j > J_1$ 时, 有 $|g(Z, \xi) - f_{t_j}(\phi(Z, \xi))| < \varepsilon, (Z, \xi) \in K$. 而 $\{f_{t_j}\}$ 在紧子集 K 上也一致收敛于 0, 则存在 $J_2 > 0$, 当 $j > J_2$ 时, 有 $|f_{t_j}(Z, \xi)| < \varepsilon, (Z, \xi) \in K$. 取 $J = \max\{J_1, J_2\}$, 当 $j > J$ 时, 令 $E = K \cap \phi(K)$, 显然, E 是 HE_I 中的一个紧子集. 因此,

$$|g(Z, \xi)| < |f_{t_j}(\phi(Z, \xi))| + \varepsilon < \varepsilon + \varepsilon = 2\varepsilon, \quad (Z, \xi) \in E.$$

由 ε 的任意性可知, $g(Z, \xi) = 0, (Z, \xi) \in E$. 再由解析函数唯一性定理, 则在 HE_I 中, $g \equiv 0$. 从而可得 $\lim_{j \rightarrow \infty} \|C_\phi f_{t_j}\|_{\beta^v} = 0$. 所得结论与假设矛盾.

(ii)→(i). 若 $\{f_t\}$ 是 $\beta^u(\text{HE}_I)$ 中的有界数列, 则 $\{f_t\}$ 在 HE_I 上是局部一致有界的. 因此, 存在子列 $\{f_{t_j}\} \subset \{f_t\}$ 在 HE_I 上内闭一致收敛, 不妨设收敛于 f , 则 $\{f_{t_j} - f\}$ 在 HE_I 上内闭一致收敛于 0, 则 $\lim_{j \rightarrow \infty} \|C_\phi(f_{t_j} - f)\|_{\beta^v} = \lim_{j \rightarrow \infty} \|C_\phi f_{t_j} - C_\phi f\|_{\beta^v} = 0$. 故 $C_\phi: \beta^u(\text{HE}_I) \rightarrow \beta^v(\text{HE}_I)$ 是紧算子. $\square$

引理 2.6 假设 $(Z, \xi), (S, t) \in \text{HE}_I$, $p_k \geq 1$, $k = 1, \dots, r$, 则存在常数 $C > 0$, 使得

$$\sum_{\substack{1 \leq \alpha \leq m \\ 1 \leq \beta \leq n}} |\det(I - Z\bar{S}^T) \text{tr}[(I - Z\bar{S}^T)^{-1} I_{\alpha\beta} \bar{S}^T]|^2 + \sum_{1 \leq k \leq r} |p_k \langle \xi_k, t_k \rangle^{p_k-1} \bar{t}_k|^2 \leq C.$$

证明 按照文献 [22, 引理 2.5] 以及考虑到 $(Z, \xi), (S, t) \in \text{HE}_I$, 容易证明本引理. $\square$

3 u -Bloch 空间到 v -Bloch 空间复合算子 C_ϕ 的有界性

定理 3.1 假设

$$H(Z, \xi; W, \eta; |\phi'(Z, \xi)|) = O(1), \quad (Z, \xi) \in \text{HE}_I, \quad (W, \eta) = \phi(Z, \xi) \rightarrow \partial \text{HE}_I, \quad (3.1)$$

则 $C_\phi: \beta^u(\text{HE}_I) \rightarrow \beta^v(\text{HE}_I)$ 有界. 反之, 若 $C_\phi: \beta^u(\text{HE}_I) \rightarrow \beta^v(\text{HE}_I)$ 有界, 则

$$H(Z, \xi; W, \eta; G(Z, \xi)) = O(1), \quad (Z, \xi) \in \text{HE}_I, \quad (W, \eta) = \phi(Z, \xi) \rightarrow \partial \text{HE}_I, \quad (3.2)$$

其中 $G(Z, \xi) = |\phi'(Z, \xi)A(Z, \xi)|$, $A(Z, \xi) = (A_{11}, \dots, A_{mn}, B_1, \dots, B_r)$, $A_{\alpha\beta} = \det(I - W\bar{W}^T) \text{tr}[(I - W\bar{W}^T)^{-1} I_{\alpha\beta} \bar{W}^T]$, $B_k = p_k |\eta_k|^{2p_k-2} \bar{\eta}_k$, $p_k \geq 1$, $\alpha = 1, \dots, m$, $\beta = 1, \dots, n$, $k = 1, \dots, r$,

$$H(Z, \xi; W, \eta; G(Z, \xi)) := \frac{[\det(I - Z\bar{Z}^T) - \sum_{j=1}^r |\xi_j|^{2p_j}]^v}{[\det(I - W\bar{W}^T) - \sum_{j=1}^r |\eta_j|^{2p_j}]^u} G(Z, \xi).$$

证明 我们知道 $\nabla f(Z, \xi) = (\frac{\partial f(Z, \xi)}{\partial z_{11}}, \dots, \frac{\partial f(Z, \xi)}{\partial z_{mn}}, \frac{\partial f(Z, \xi)}{\partial \xi_1}, \dots, \frac{\partial f(Z, \xi)}{\partial \xi_r})$. 所以,

$$\begin{aligned} |\nabla(f \circ \phi)(Z, \xi)|^2 &= \sum_{\substack{1 \leq \alpha \leq m \\ 1 \leq \beta \leq n}} \left| \sum_{\substack{1 \leq k \leq m \\ 1 \leq l \leq n}} \frac{\partial f(\phi(Z, \xi))}{\partial X_{kl}} \frac{\partial \psi_{kl}(Z, \xi)}{\partial z_{\alpha\beta}} + \sum_{1 \leq j \leq r} \frac{\partial f(\phi(Z, \xi))}{\partial Y_j} \frac{\partial \varphi_j(Z, \xi)}{\partial z_{\alpha\beta}} \right|^2 \\ &\quad + \sum_{1 \leq i \leq r} \left| \sum_{\substack{1 \leq k \leq m \\ 1 \leq l \leq n}} \frac{\partial f(\phi(Z, \xi))}{\partial X_{kl}} \frac{\partial \psi_{kl}(Z, \xi)}{\partial \xi_i} + \sum_{1 \leq j \leq r} \frac{\partial f(\phi(Z, \xi))}{\partial Y_j} \frac{\partial \varphi_j(Z, \xi)}{\partial \xi_i} \right|^2 \\ &\leq (mn + r) \left[\sum_{\substack{1 \leq k \leq m \\ 1 \leq l \leq n}} \left| \frac{\partial f(\phi(Z, \xi))}{\partial X_{kl}} \right|^2 + \sum_{1 \leq j \leq r} \left| \frac{\partial f(\phi(Z, \xi))}{\partial Y_j} \right|^2 \right] \\ &\quad \times \left\{ \sum_{\substack{1 \leq \alpha \leq m \\ 1 \leq \beta \leq n}} \left[\sum_{\substack{1 \leq k \leq m \\ 1 \leq l \leq n}} \left| \frac{\partial \psi_{kl}(Z, \xi)}{\partial z_{\alpha\beta}} \right|^2 + \sum_{1 \leq j \leq r} \left| \frac{\partial \varphi_j(Z, \xi)}{\partial z_{\alpha\beta}} \right|^2 \right] \right. \\ &\quad \left. + \sum_{1 \leq i \leq r} \left[\sum_{\substack{1 \leq k \leq m \\ 1 \leq l \leq n}} \left| \frac{\partial \psi_{kl}(Z, \xi)}{\partial \xi_i} \right|^2 + \sum_{1 \leq j \leq r} \left| \frac{\partial \varphi_j(Z, \xi)}{\partial \xi_i} \right|^2 \right] \right\} \\ &= (mn + r) |\nabla f(\phi(Z, \xi))|^2 |\phi'(Z, \xi)|^2. \end{aligned}$$

设 $f \in \beta^u(\text{HE}_I)$, $(Z, \xi) \in \text{HE}_I$, 结合引理 2.1, 我们有

$$\begin{aligned} & \left[\det(I - Z\bar{Z}^T) - \sum_{j=1}^r |\xi_j|^{2p_j} \right]^v |\nabla C_\phi f(Z, \xi)| \\ & \leq \sqrt{mn+r} \left[\det(I - Z\bar{Z}^T) - \sum_{j=1}^r |\xi_j|^{2p_j} \right]^v |\nabla f(\phi(Z, \xi))| |\phi'(Z, \xi)| \\ & \leq C\sqrt{mn+r} |\nabla f(\phi(Z, \xi))| \left[\det(I - W\bar{W}^T) - \sum_{j=1}^r |\eta_j|^{2p_j} \right]^u \\ & \leq C\sqrt{mn+r} \|f\|_{\beta^u}. \end{aligned}$$

再由引理 2.3, 我们有

$$\|C_\phi f\|_v = |f(\phi(0, 0))| + \sup \left[\det(I - Z\bar{Z}^T) - \sum_{j=1}^r |\xi_j|^{2p_j} \right]^v |\nabla C_\phi f(Z, \xi)| \leq C\|f\|_u.$$

所以, $C_\phi : \beta^u(\text{HE}_I) \rightarrow \beta^v(\text{HE}_I)$ 有界.

反之, 若 $C_\phi : \beta^u(\text{HE}_I) \rightarrow \beta^v(\text{HE}_I)$ 是有界算子, 则对任意 $f \in \beta^u(\text{HE}_I)$, 存在常数 $C > 0$, 使得

$$\|C_\phi f\|_v \leq C\|f\|_u. \quad (3.3)$$

当 $u \neq \frac{1}{2}$ 时, $(S, t) \in \text{HE}_I$, 构造 $\beta^u(\text{HE}_I)$ 上的函数

$$f_{(S,t)}(Z, \xi) = \frac{1}{2u-1} \left\{ \frac{1}{[\det(I - Z\bar{S}^T) - \sum_{j=1}^r \langle \xi_j, t_j \rangle^{p_j}]^{2u-1}} - 1 \right\}.$$

于是,

$$\begin{aligned} \frac{\partial f_{(S,t)}}{\partial z_{\alpha\beta}} &= \frac{\det(I - Z\bar{S}^T) \text{tr}[(I - Z\bar{S}^T)^{-1} I_{\alpha\beta} \bar{S}^T]}{[\det(I - Z\bar{S}^T) - \sum_{j=1}^r \langle \xi_j, t_j \rangle^{p_j}]^{2u}}, \quad \alpha = 1, \dots, m, \quad \beta = 1, \dots, n, \\ \frac{\partial f_{(S,t)}}{\partial \xi_k} &= \frac{p_k \langle \xi_k, t_k \rangle^{p_k-1} \bar{t}_k}{[\det(I - Z\bar{S}^T) - \sum_{j=1}^r \langle \xi_j, t_j \rangle^{p_j}]^{2u}}, \quad k = 1, \dots, r. \end{aligned}$$

因此, 结合引理 2.4 和 2.6, 我们有

$$\begin{aligned} & \left[\det(I - Z\bar{Z}^T) - \sum_{j=1}^r |\xi_j|^{2p_j} \right]^u |\nabla f_{(S,t)}(Z, \xi)| \\ &= \left[\det(I - Z\bar{Z}^T) - \sum_{j=1}^r |\xi_j|^{2p_j} \right]^u \\ & \quad \times \frac{\left\{ \sum_{\substack{1 \leq \alpha \leq m \\ 1 \leq \beta \leq n}} |\det(I - Z\bar{S}^T) \text{tr}[(I - Z\bar{S}^T)^{-1} I_{\alpha\beta} \bar{S}^T]|^2 + \sum_{1 \leq k \leq r} |p_k \langle \xi_k, t_k \rangle^{p_k-1} \bar{t}_k|^2 \right\}^{\frac{1}{2}}}{|\det(I - Z\bar{S}^T) - \sum_{j=1}^r \langle \xi_j, t_j \rangle^{p_j}|^{2u}} \\ & \leq \frac{\left\{ \sum_{\substack{1 \leq \alpha \leq m \\ 1 \leq \beta \leq n}} |\det(I - Z\bar{S}^T) \text{tr}[(I - Z\bar{S}^T)^{-1} I_{\alpha\beta} \bar{S}^T]|^2 + \sum_{1 \leq k \leq r} |p_k \langle \xi_k, t_k \rangle^{p_k-1} \bar{t}_k|^2 \right\}^{\frac{1}{2}}}{[\det(I - S\bar{S}^T) - \sum_{j=1}^r |t_j|^{2p_j}]^u} \\ & \leq \frac{C}{[\det(I - S\bar{S}^T) - \sum_{j=1}^r |t_j|^{2p_j}]^u}. \end{aligned}$$

所以,

$$\|f_{(S,t)}(Z, \xi)\|_u \leq \frac{C}{[\det(I - S\bar{S}^T) - \sum_{j=1}^r |t_j|^{2p_j}]^u}. \quad (3.4)$$

另一方面, 因为

$$\begin{aligned} \frac{\partial f_{(S,t)}(\phi(Z, \xi))}{\partial X_{kl}} &= \frac{\det(I - W\bar{S}^T) \operatorname{tr}[(I - W\bar{S}^T)^{-1} I_{kl} \bar{S}^T]}{[\det(I - W\bar{S}^T) - \sum_{j=1}^r \langle \eta_j, t_j \rangle^{p_j}]^{2u}}, \quad k = 1, \dots, m, \quad l = 1, \dots, n, \\ \frac{\partial f_{(S,t)}(\phi(Z, \xi))}{\partial Y_j} &= \frac{p_j \langle \eta_j, t_j \rangle^{p_j-1} \bar{t}_j}{[\det(I - W\bar{S}^T) - \sum_{j=1}^r \langle \eta_j, t_j \rangle^{p_j}]^{2u}}, \quad j = 1, \dots, r, \end{aligned}$$

因此,

$$\begin{aligned} & \left[\det(I - Z\bar{Z}^T) - \sum_{j=1}^r |\xi_j|^{2p_j} \right]^v |\nabla(C_\phi f_{(S,t)})(Z, \xi)| \\ &= \frac{[\det(I - Z\bar{Z}^T) - \sum_{j=1}^r |\xi_j|^{2p_j}]^v}{|\det(I - W\bar{S}^T) - \sum_{j=1}^r \langle \eta_j, t_j \rangle^{p_j}|^{2u}} \\ & \times \left\{ \sum_{\substack{1 \leq \alpha \leq m \\ 1 \leq \beta \leq n}} \left| \sum_{\substack{1 \leq k \leq m \\ 1 \leq l \leq n}} \det(I - W\bar{S}^T) \operatorname{tr}[(I - W\bar{S}^T)^{-1} I_{kl} \bar{S}^T] \frac{\partial \psi_{kl}(Z, \xi)}{\partial z_{\alpha\beta}} \right. \right. \\ & + \sum_{1 \leq j \leq r} p_j \langle \eta_j, t_j \rangle^{p_j-1} \bar{t}_j \frac{\partial \varphi_j(Z, \xi)}{\partial z_{\alpha\beta}} \left. \right|^2 \\ & + \sum_{1 \leq i \leq r} \left| \sum_{\substack{1 \leq k \leq m \\ 1 \leq l \leq n}} \det(I - W\bar{S}^T) \operatorname{tr}[(I - W\bar{S}^T)^{-1} I_{kl} \bar{S}^T] \frac{\partial \psi_{kl}(Z, \xi)}{\partial \xi_i} \right. \\ & \left. \left. + \sum_{1 \leq j \leq r} p_j \langle \eta_j, t_j \rangle^{p_j-1} \bar{t}_j \frac{\partial \varphi_j(Z, \xi)}{\partial \xi_i} \right|^2 \right\}^{\frac{1}{2}}. \end{aligned}$$

令 $(S, t) = (W, \eta) = \phi(Z, \xi)$, 则由上式、(3.3) 和 (3.4), 并整理即可得到 $H(Z, \xi; W, \eta; G(Z, \xi)) \leq C$. 当 $u = \frac{1}{2}$ 时, 我们定义 $f_{(S,t)}(Z, \xi) = \ln \frac{1}{\det(I - Z\bar{S}^T) - \sum_{j=1}^r \langle \xi_j, t_j \rangle^{p_j}}$, 则同样可以证明定理结论成立. $\square$

4 u -Bloch 空间到 v -Bloch 空间复合算子 C_ϕ 的紧性

定理 4.1 假设

$$H(Z, \xi; W, \eta; |\phi'(Z, \xi)|) = o(1), \quad (Z, \xi) \in \operatorname{HE}_I, \quad (W, \eta) = \phi(Z, \xi) \rightarrow \partial \operatorname{HE}_I, \quad (4.1)$$

则 $C_\phi: \beta^u(\operatorname{HE}_I) \rightarrow \beta^v(\operatorname{HE}_I)$ 是紧算子. 反之, 若 $C_\phi: \beta^u(\operatorname{HE}_I) \rightarrow \beta^v(\operatorname{HE}_I)$ 是紧算子, 则

$$H(Z, \xi; W, \eta; G(Z, \xi)) = o(1), \quad (Z, \xi) \in \operatorname{HE}_I, \quad (W, \eta) = \phi(Z, \xi) \rightarrow \partial \operatorname{HE}_I, \quad (4.2)$$

其中 $p_j \geq 1, j = 1, \dots, r, G(Z, \xi), |\phi'(Z, \xi)|$ 等和定理 3.1 中相同.

证明 充分性 设 $\{f_t\}$ 是 $\beta^u(\operatorname{HE}_I)$ 中的有界函数列, $\|f_t\|_{\beta^u} \leq C$, 且 $\{f_t\}$ 在 HE_I 的紧子集上一致收敛于 0. 进一步, 由 Weierstrass 定理可知, $\{\nabla f_t\}$ 在 HE_I 的紧子集上也一致收敛于 0.

由 (4.1) 知, $\forall \varepsilon > 0$, 存在 $\sigma > 0$, 当 $\operatorname{dist}((W, \eta), \partial \operatorname{HE}_I) < \sigma$ 时,

$$H(Z, \xi; W, \eta; |\phi'(Z, \xi)|) < \varepsilon. \quad (4.3)$$

用定理 (3.1) 中相同的方法知,

$$\begin{aligned} & \left[\det(I - Z\bar{Z}^T) - \sum_{j=1}^r |\xi_j|^{2p_j} \right]^v |\nabla(C_\phi f_t)(Z, \xi)| \\ & \leq \left[\det(I - Z\bar{Z}^T) - \sum_{j=1}^r |\xi_j|^{2p_j} \right]^v \sqrt{mn+r} |\nabla f_t(\phi(Z, \xi))| |\phi'(Z, \xi)| \\ & \leq \varepsilon \sqrt{mn+r} \|f_t\|_{\beta^u} \leq \varepsilon \sqrt{mn+r} \cdot C. \end{aligned}$$

令 $E_\sigma = \{(W, \eta) \in \text{HE}_I, \text{dist}((W, \eta), \partial\text{HE}_I) \geq \sigma\}$, 则 E_σ 是 HE_I 中的紧子集.

令 $(W, \eta) \in E_\sigma$, 则由引理 2.1 的证明知, 存在常数 C , 使得 $H(Z, \xi; W, \eta; |\phi'(Z, \xi)|) < C$. 于是,

$$\left[\det(I - Z\bar{Z}^T) - \sum_{j=1}^r |\xi_j|^{2p_j} \right]^v |\nabla(C_\phi f_t)(Z, \xi)| \leq \sqrt{mn+r} \cdot C |\nabla f_t(\phi(Z, \xi))|.$$

所以, 当 $t \rightarrow \infty$ 时,

$$\|C_\phi f_t\|_u = |f_t(\phi(0, 0))| + \sup \left[\det(I - Z\bar{Z}^T) - \sum_{j=1}^r |\xi_j|^{2p_j} \right]^u |\nabla(f_t \circ \phi)(Z, \xi)| \rightarrow 0.$$

故 $C_\phi : \beta^u(\text{HE}_I) \rightarrow \beta^v(\text{HE}_I)$ 是紧的.

必要性 下面用反证法. 假设结论不成立, 则存在 $\varepsilon_0 > 0$, 且在 HE_I 上存在一序列 $\{Z^i, \xi^i\}$, 满足 $\phi(Z^i, \xi^i) \rightarrow \partial\text{HE}_I, i \rightarrow \infty$, 使得

$$\frac{[\det(I - Z^i \bar{Z}^i{}^T) - \sum_{j=1}^r |\xi_j^i|^{2p_j}]^v}{[\det(I - W^i \bar{W}^i{}^T) - \sum_{j=1}^r |\eta_j^i|^{2p_j}]^u} G(Z^i, \xi^i) \geq \varepsilon_0, \quad i = 1, 2, \dots, \quad (4.4)$$

这里 $(W^i, \eta^i) = \phi(Z^i, \xi^i), i = 1, 2, \dots$.

下面构造函数列 $\{f_i(Z, \xi)\}$ 满足下列三个条件:

- (i) $\{f_i(Z, \xi)\}$ 是 $\beta^u(\text{HE}_I)$ 中的有界函数列;
- (ii) $\{f_i(Z, \xi)\}$ 在 HE_I 的紧子集上一致收敛于 0;
- (iii) $\|C_\phi f_i(Z, \xi)\|_{\beta^v} \rightarrow 0, i \rightarrow \infty$,

则由引理 2.5 知, 这与 C_ϕ 的紧性相矛盾, 从而证明 (4.2) 是必要条件.

当 $u \neq \frac{1}{2}$ 时, 构造函数列

$$f_i(Z, \xi) = \frac{1}{2u-1} \frac{[\det(I - W^i \bar{W}^i{}^T) - \sum_{j=1}^r |\eta_j^i|^{2p_j}]^u}{[\det(I - Z\bar{W}^i{}^T) - \sum_{j=1}^r \langle \xi_j, \eta_j^i \rangle^{p_j}]^{2u-1}}.$$

结合引理 2.4 和 2.6 有

$$\begin{aligned} & \left[\det(I - Z\bar{Z}^T) - \sum_{j=1}^r |\xi_j|^{2p_j} \right]^u |\nabla f_i(Z, \xi)| \\ & = \frac{[\det(I - Z\bar{Z}^T) - \sum_{j=1}^r |\xi_j|^{2p_j}]^u [\det(I - W^i \bar{W}^i{}^T) - \sum_{j=1}^r |\eta_j^i|^{2p_j}]^u}{[\det(I - Z\bar{W}^i{}^T) - \sum_{j=1}^r \langle \xi_j, \eta_j^i \rangle^{p_j}]^{2u}} \end{aligned}$$

$$\begin{aligned} & \times \left\{ \sum_{\substack{1 \leq \alpha \leq m \\ 1 \leq \beta \leq n}} |\det(I - Z\bar{W}^i)^T \text{tr}[(I - Z\bar{W}^i)^{-1} I_{\alpha\beta} \bar{W}^i]^T]|^2 + \sum_{1 \leq k \leq r} |p_k \langle \xi_k, \eta_k^i \rangle^{p_k-1} \bar{\eta}_k^i|^2 \right\}^{\frac{1}{2}} \\ & \leq \left\{ \sum_{\substack{1 \leq \alpha \leq m \\ 1 \leq \beta \leq n}} |\det(I - Z\bar{W}^i)^T \text{tr}[(I - Z\bar{W}^i)^{-1} I_{\alpha\beta} \bar{W}^i]^T]|^2 + \sum_{1 \leq k \leq r} |p_k \langle \xi_k, \eta_k^i \rangle^{p_k-1} \bar{\eta}_k^i|^2 \right\}^{\frac{1}{2}} \leq C. \end{aligned}$$

故 $\{f_i(Z, \xi)\}$ 是 $\beta^u(\text{HE}_I)$ 中的有界函数列. 满足条件 (i).

由引理 2.4 可知,

$$|f_i(Z, \xi)| \leq \frac{1}{2u-1} \frac{[\det(I - W^i \bar{W}^i)^T - \sum_{j=1}^r |\eta_j^i|^{2p_j}]^{\frac{1}{2}}}{[\det(I - Z\bar{Z}^T) - \sum_{j=1}^r |\xi_j|^{2p_j}]^{u-\frac{1}{2}}}.$$

当 $i \rightarrow \infty$ 时, $(W^i, \eta^i) \rightarrow \partial \text{HE}_I$, 从而, $\det(I - W^i \bar{W}^i)^T - \sum_{j=1}^r |\eta_j^i|^{2p_j} \rightarrow 0, i \rightarrow \infty$. 而 $(Z, \xi) \in K, K$ 为 HE_I 的紧子集, 则 $\det(I - Z\bar{Z}^T) - \sum_{j=1}^r |\xi_j|^{2p_j}$ 存在正的下界. 从而, $\{f_i(Z, \xi)\}$ 在 HE_I 的紧子集上一致收敛于 0, 即 $\{f_i(Z, \xi)\}$ 满足条件 (ii).

经直接计算, 并结合 (4.4) 知,

$$\begin{aligned} \left[\det(I - Z^i \bar{Z}^i)^T - \sum_{j=1}^r |\xi_j^i|^{2p_j} \right]^v |\nabla(C_\phi f_i)(Z^i, \xi^i)| &= \frac{[\det(I - Z^i \bar{Z}^i)^T - \sum_{j=1}^r |\xi_j^i|^{2p_j}]^v}{[\det(I - W^i \bar{W}^i)^T - \sum_{j=1}^r |\eta_j^i|^{2p_j}]^u} G(Z^i, \xi^i) \\ &\geq \varepsilon_0, \quad i = 1, 2, \dots \end{aligned}$$

由此可知, $i \rightarrow \infty$ 时, $\|C_\phi f_i\|_{\beta^v} \rightarrow 0$, 即 $\{f_i(Z, \xi)\}$ 满足条件 (iii).

当 $u = \frac{1}{2}$ 时, 可设函数列

$$f_i(Z, \xi) = \left[\det(I - W^i \bar{W}^i)^T - \sum_{j=1}^r |\eta_j^i|^{2p_j} \right]^{\frac{1}{2}} \ln \left[\det(I - Z\bar{W}^i)^T - \sum_{j=1}^r \langle \xi_j, \eta_j^i \rangle^{p_j} \right], \quad i = 1, 2, \dots,$$

则

$$\begin{aligned} \frac{\partial f_i(Z, \xi)}{\partial z_{\alpha\beta}} &= \frac{[\det(I - W^i \bar{W}^i)^T - \sum_{j=1}^r |\eta_j^i|^{2p_j}]^{\frac{1}{2}} \det(I - Z\bar{W}^i)^T \text{tr}[(I - Z\bar{W}^i)^{-1} I_{\alpha\beta} \bar{W}^i]^T}{\det(I - Z\bar{W}^i)^T - \sum_{j=1}^r \langle \xi_j, \eta_j^i \rangle^{p_j}}, \\ \frac{\partial f_i(Z, \xi)}{\partial \xi_g} &= \frac{[\det(I - W^i \bar{W}^i)^T - \sum_{j=1}^r |\eta_j^i|^{2p_j}]^{\frac{1}{2}} p_g \langle \xi_g, \eta_g^i \rangle^{p_g-1} \bar{\eta}_g^i}{\det(I - Z\bar{W}^i)^T - \sum_{j=1}^r \langle \xi_j, \eta_j^i \rangle^{p_j}}, \\ \alpha &= 1, 2, \dots, m, \quad \beta = 1, 2, \dots, n, \quad g = 1, \dots, r, \quad i = 1, 2, \dots \end{aligned}$$

如上类似可以证明, 函数列 $\{f_i(Z, \xi)\}$ 满足条件 (i)–(iii). 定理证毕. $\square$

致谢 感谢 LIN MingHua 博士关于证明引理 2.4 的建议. 感谢所有审稿人仔细审阅本文, 并提出细致中肯的修改意见.

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Boundedness and compactness of the composition operators from u -Bloch space to v -Bloch space on the first Hua domains

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Abstract In this paper, we introduce a weighted Bloch space and discuss the boundedness and compactness of composition operators C_ϕ from the u -Bloch space $\beta^u(\text{HE}_I)$ to the v -Bloch space $\beta^v(\text{HE}_I)$, and obtain the necessary condition and sufficient condition on the boundedness and compactness of composition operators C_ϕ on the first Hua domains.

Keywords the Hua domains, weighted Bloch space, boundedness and compactness of composition operators

MSC(2010) 32A18, 47B33

doi: 10.1360/N012015-00146