刚-柔耦合的对称天线结构非线性主共振

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摘要 本文研究了主共振激励下空间天线的非线性动态响应. 考虑机械臂与天线的耦合, 提出了一种由两级刚性臂与两根各向同性柔性梁组成的T-型梁式结构. 采用Lagrange方程和假设模态方法, 建立了T-型刚柔耦合天线面内四自由度非线性动力学方程. 利用多尺度法获得主共振三次近似解, 得出系统主共振幅频响应解析方程. 对系统幅频响应进行分析, 结果表明主共振响应存在多解和非线性跳跃现象, 激励对模态幅值及非线性特性具有显著影响. 随着激励的增大, 系统出现由刚度硬特性向软特性的转变; 当激励超过临界值时, 系统幅频响应表现出刚度渐软特性, 共振频率随简谐激励幅值的增大而减小.

关键词 刚柔耦合, 天线结构, 非线性共振, 四自由度, 假设模态

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1 引言

航天技术的迅速发展, 迫切需要掌握可入轨后展开的大型空间结构技术^[1-4]. 大型天线结构在轨展开锁定后, 形成的大尺度结构具有阻尼小、模态密集等动力学特征, 易于发生模态之间的动态耦合.

通常可基于能量等效原理将复杂天线结构简化为同构型的梁、圆环、板等一维或二维连续体柔性结构^[5], 以期把握或控制这类刚-柔耦合系统的动力学行为^[6,7]. 对于简化为梁结构的非线性动力学系统, Ji等人^[8]分析了带有集中质量的T-型梁结构在参数激励下的1:2内共振和主共振, 并证明系统存在Smale马蹄

意义下的混沌运动. Emam等人^[9-11]研究了屈曲梁在主共振/亚谐共振激励下含/不含内共振时的非线性动力学响应, 揭示了系统可能发生的鞍结分岔、Hopf分岔和非线性跳跃等复杂动力学行为. Wang等人^[12]研究了带有附加质量的三梁组合结构幅频响应的叉形分岔、鞍结分岔和Hopf分岔等现象. Gao等人^[13]分析了带有大小支撑臂的柔性梁天线结构非线性共振问题.

本文针对一种简化的T-型梁式天线结构, 建立其四自由度非线性动力学方程, 通过多尺度方法分析刚性臂与柔性天线之间存在的非线性共振, 以揭示这类对称天线结构在一阶主共振下的非线性动态响应特征.

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2 天线结构模型

在卫星本体坐标系下, 考察图1所示对称天线结构的非线性共振问题. 该天线结构由等效梁结构和两根刚性支撑臂组成. 刚性臂长度和质量分别为 l_i 和 m_i , 转动惯量为 J_i , $i=1, 2$. 第一级刚性臂与卫星本体间的联结刚度为 k_1 , 并通过刚度 k_2 连接到第二级刚性臂. 柔性梁中部 O 点固结于第二级刚性臂, 形成一种T-型对称结构. 考虑天线为Euler-Bernoulli梁结构, 质量密度为 ρ 、横截面积 A 、长度 L 、抗弯刚度 EI .

建立图1所示惯性坐标系 OXY 和连体坐标系 ox_1y_1 和 ox_2y_2 . 连体坐标系到惯性坐标系的转换矩阵分别为

$$A_0 = \begin{bmatrix} \cos(\theta_1 + \theta_2 - \pi/2) & -\sin(\theta_1 + \theta_2 - \pi/2) \\ \sin(\theta_1 + \theta_2 - \pi/2) & \cos(\theta_1 + \theta_2 - \pi/2) \end{bmatrix}, \quad (1)$$

$$B_0 = \begin{bmatrix} \cos(\theta_1 + \theta_2 + \pi/2) & -\sin(\theta_1 + \theta_2 + \pi/2) \\ \sin(\theta_1 + \theta_2 + \pi/2) & \cos(\theta_1 + \theta_2 + \pi/2) \end{bmatrix}. \quad (2)$$

不考虑重力势能, 系统动能和势能分别为

$$T = \frac{1}{2} \sum_{i=1}^2 m_i \dot{\mathbf{r}}_i^T \dot{\mathbf{r}}_i + \frac{1}{2} (J_1 \dot{\theta}_1^2 + J_2 (\dot{\theta}_1 + \dot{\theta}_2)^2) + \frac{1}{2} \int_0^L \rho A (\dot{\mathbf{r}}_p^T \dot{\mathbf{r}}_p + \dot{\mathbf{r}}_q^T \dot{\mathbf{r}}_q) dx, \quad (3)$$

$$V = \frac{1}{2} \sum_{i=1}^2 k_i \theta_i^2 + \frac{1}{2} \int_0^L EI \sum_{i=1}^2 \left(\frac{\partial^2 w_i}{\partial x_i^2} \right)^2 dx_i, \quad (4)$$

图1 (网络版彩图)星载天线系统简化模型

Figure 1 (Color online) The simplified model of a satellite-antenna system.

式中, $\mathbf{r}_1$ 和 $\mathbf{r}_2$ 分别为两个刚性臂质心的位矢, $\mathbf{r}_p$ 和 $\mathbf{r}_q$ 为相应梁上任一点的位矢. 假设刚性臂转动为低转速, 柔性梁为小变形, 基于假设模态法, 梁的挠度表示为

$$w_i(x_i, t) = \varphi(x_i)q_i(t), \quad i = 1, 2, \quad (5)$$

式中, $\varphi(x_i)$ 为悬臂梁第一阶模态函数, $q_i(t)$ 为相应的模态坐标.

选取广义坐标 $\boldsymbol{\eta} = [\theta_1 \theta_2 q_1 q_2]^T = [\eta_1 \eta_2 \eta_3 \eta_4]^T$, 经Lagrange方程推导出系统动力学方程为

$$\mathbf{M}(\boldsymbol{\eta})\ddot{\boldsymbol{\eta}} + \mathbf{K}\boldsymbol{\eta} - \mathbf{P}(\boldsymbol{\eta}, \dot{\boldsymbol{\eta}}) = 0, \quad (6)$$

式中, $\mathbf{M}(\boldsymbol{\eta})$ 和 $\mathbf{K}$ 分别为质量矩阵和刚度矩阵, $\mathbf{P}(\boldsymbol{\eta}, \dot{\boldsymbol{\eta}})$ 为离心力与科氏力之和向量, 具体见附录.

假设连接两个刚性臂的联结关节受到与电机输入电压平方成正比的力矩电机所驱动. 驱动力矩为

$$\boldsymbol{\tau} = v^2(t)\mathbf{L}, \quad (7)$$

其中, $v(t)$ 为驱动电机的电压, $\mathbf{L} = [0 \ c \ 0 \ 0]^T$ 为常数列向量, c 为与电机特性相关的常数. 此时方程(6)成为

$$\mathbf{M}(\boldsymbol{\eta})\ddot{\boldsymbol{\eta}} + \mathbf{K}\boldsymbol{\eta} - \mathbf{P}(\boldsymbol{\eta}, \dot{\boldsymbol{\eta}}) = \boldsymbol{\tau}. \quad (8)$$

3 主共振分析

设电机输入电压为

$$v(t) = v_0 + v_3 \cos \omega_f t, \quad (9)$$

其中, v_0 和 v_3 分别为直流电压和交流电压. 令频率 ω_f 为

$$\omega_f = \omega_1 + \sigma, \quad (10)$$

调谐参数 σ 代表激励频率与系统第一阶固有频率的偏离程度.

采用多尺度法^[14,15], 设解为

$$\boldsymbol{\eta}(t, \varepsilon) = \boldsymbol{\eta}_0 + \varepsilon \boldsymbol{\eta}_1(T_0, T_1, T_2) + \varepsilon^2 \boldsymbol{\eta}_2(T_0, T_1, T_2) + \varepsilon^3 \boldsymbol{\eta}_3(T_0, T_1, T_2) + O(\varepsilon^4), \quad (11)$$

其中, $T_r = \varepsilon^r t$, $r = 0, 1, 2, \dots$, ε 为任意小参数. 为了使非线性平衡激励的影响, 引入尺度变量

$$v_3 \rightarrow \varepsilon^3 v_3, \quad \sigma \rightarrow \varepsilon^2 \sigma, \quad (12)$$

将式(9)改写为

$$v(t) = v_0 + \frac{1}{2} \varepsilon^3 v_3 (e^{i\sigma T_2} e^{i\omega_1 T_0} + cc), \quad (13)$$

其中, cc 表示前面各项的共轭. 于是有

$$v^2(t)L = v_0^2L + \varepsilon^3 v_0 v_3 (e^{i\omega_1 T_2} e^{i\omega_1 T_0} + cc)L + O(\varepsilon^4). \quad (14)$$

将式(11)和(14)代入方程(8), 比较 ε 同次幂系数, 得

$$K_0 \eta_0 - P_0 - v_0^2 L = 0, \quad (15)$$

$$M_0 D_0^2 \eta_1 + R_0 \eta_1 = 0, \quad (16)$$

$$\begin{aligned} M_0 D_0^2 \eta_2 + R_0 \eta_2 = & -2M_0 D_0 D_1 \eta_1 - K_1(\eta_1) \eta_1 \\ & -M_1(\eta_1) D_0^2 \eta_1 - K_2(\eta_1, \eta_1) \eta_0 \\ & + P_2(\eta_1, D_0 \eta_1) + P_{12} D_1 \eta_1, \end{aligned} \quad (17)$$

$$\begin{aligned} M_0 D_0^2 \eta_3 + R_0 \eta_3 = & -2M_0 D_0 D_2 \eta_1 - M_0 D_1^2 \eta_1 \\ & -2M_0 D_0 D_1 \eta_2 - 2M_1(\eta_1) D_0 D_1 \eta_1 \\ & -K_1(\eta_1) \eta_2 - M_1(\eta_1) D_0^2 \eta_2 \\ & -2K_2(\eta_1, \eta_2) \eta_0 - K_1(\eta_2) \eta_1 \\ & -K_2(\eta_1, \eta_1) \eta_1 - M_2(\eta_1, \eta_1) D_0^2 \eta_1 \\ & + P_{12} D_1 \eta_2 + 2P_{21}(\eta_1, D_0 \eta_2 + D_1 \eta_1) \\ & -K_3(\eta_1, \eta_1, \eta_1) \eta_0 - M_1(\eta_2) D_0^2 \eta_1 \\ & + P_{12} D_2 \eta_1 + 2P_{21}(\eta_2, D_0 \eta_1) \\ & + P_3(\eta_1, D_0 \eta_1) + v_0 v_3 (e^{i\omega_1 T_2} e^{i\omega_1 T_0} + cc)L, \end{aligned} \quad (18)$$

其中,

$$D_r = \frac{\partial}{\partial T_r}, \quad \mathbf{0} = [0000]^T,$$

$$M_0 = M(\eta_0), \quad K_0 = K(\eta_0), \quad P_0 = P(\eta_0, D_0 \eta_0), \quad (19)$$

$$\begin{cases} M_1(x) = \left[\frac{\partial M}{\partial \eta_i} x_i \right], \quad K_1(x) = \left[\frac{\partial K}{\partial \eta_i} x_i \right], \\ P_{11} = [P_{11ij}] = \left[\frac{\partial P_i}{\partial \eta_j} \right], \quad P_{12} = [P_{12ij}] = \left[\frac{\partial P_i}{\partial \eta_j} \right], \end{cases} \quad (20)$$

$$\begin{cases} M_2(x, y) = \left[\frac{1}{2} \frac{\partial^2 M}{\partial \eta_i \partial \eta_j} x_i y_j \right], \\ K_2(x, y) = \left[\frac{1}{2} \frac{\partial^2 K}{\partial \eta_i \partial \eta_j} x_i y_j \right], \\ P_2(x, y) = [P_{2k}(x, y)] = \left[\frac{1}{2} \left(\frac{\partial P_k}{\partial \eta_i} x_i + \frac{\partial P_k}{\partial \eta_j} y_j \right)^2 \right], \\ P_{21}(x, y) = [P_{21k}(x, y)] = \left[\frac{1}{2} \frac{\partial^2 P_k}{\partial \eta_i \partial \eta_j} x_i y_j \right], \end{cases} \quad (21)$$

$$\begin{cases} P_3(x, y) = [P_{3k}(x, y)] = \left[\frac{1}{6} \left(\frac{\partial P_k}{\partial \eta_i} x_i + \frac{\partial P_k}{\partial \eta_j} y_j \right)^3 \right], \\ K_3(x, y, z) = \left[\frac{1}{6} \frac{\partial^3 K}{\partial \eta_i \partial \eta_j \partial \eta_k} x_i y_j z_k \right], \end{cases} \quad (22)$$

$$\begin{cases} R_0 = K_0 - P_{11} - P_{12} D_0 + Q_0, \\ Q_0 = \left[\left\{ \frac{\partial K}{\partial \eta_i} \eta_0 \right\} \left\{ \frac{\partial K}{\partial \eta_2} \eta_0 \right\} \cdots \left\{ \frac{\partial K}{\partial \eta_N} \eta_0 \right\} \right]_{\eta_0}, \end{cases} \quad (23)$$

其中, v_i 表示向量 $\mathbf{v}$ 的第 i 个分量, 所有偏导数均取在 $\eta = \eta_0$ 的值且应用Einstein求和约定.

ε^0 : 对于方程(15), 求得

$$\eta_0 = K_0^{-1}(P_0 + v_0^2 L).$$

ε^1 : 对于方程(16), 若 M_0 非奇异, 则

$$D_0^2 \eta_1 + S_0 \eta_1 = 0, \quad (24)$$

式中, $S_0 = M_0^{-1} R_0$ 为关于 η_0 的函数矩阵. 求解方程(24)得一次近似解

$$\eta_1 = \sum_{j=1}^N (A_j(T_1, T_2) e^{i\omega_j T_0} \mathbf{p}_j + cc), \quad (25)$$

这里特征值 ω_j^2 和对应的特征向量 $\mathbf{p}_j$ 分别代表在 η_0 附近的频率和振型, 满足

$$S_0 \mathbf{p}_j = \omega_j^2 \mathbf{p}_j (j = 1, 2, \dots, N). \quad (26)$$

ε^2 : 将 η_0, η_1 表达式代入方程(17), 消除永年项, 得

$$D_1 A_j(T_1, T_2) = 0, \quad A_j(T_1, T_2) = A_j(T_2). \quad (27)$$

于是二次近似解为

$$\begin{aligned} \eta_2 = & A(\omega) [-M_1(\eta_1) D_0^2 \eta_1 - K_2(\eta_1, \eta_1) \eta_0 \\ & - K_1(\eta_1) \eta_1 + P_2(\eta_1, D_0 \eta_1)], \end{aligned} \quad (28)$$

其中,

$$\Lambda(\omega) = A^{-1}(\omega) = (R_0 - M_0 \omega^2)^{-1}$$

为匹配方程(28)右端谐波项中各阶频率的矩阵.

ε^3 : 将 η_0, η_1, η_2 表达式代入方程(18), 当永年项与 $\mathbf{u}_j$ 正交, 则方程(18)存在一致解 η_3 . $\mathbf{u}_j$ 满足

$$\omega_j^2 M_0^T \mathbf{u}_j = R_0^T \mathbf{u}_j. \quad (29)$$

本文只考虑一阶主共振响应且不包含其他模态的内共振. 通过消除永年项, 有

$$\mathbf{u}_1^T \left(\mathbf{v}_{1r} A'_1 + A_1^2 \bar{A}_1 \mathbf{v}_{12} + \sum_{k=2}^N A_1 A_k \bar{A}_k \mathbf{v}_{1k} + v_0 v_3 e^{i\omega T_2} \mathbf{L} \right) = 0, \quad (30)$$

$$\mathbf{u}_j^T \left(\mathbf{v}_{jr} A'_j + A_j^2 \bar{A}_j \mathbf{v}_{j2} + \sum_{k=1, k \neq j}^N A_j A_k \bar{A}_k \mathbf{v}_{jk} \right) = 0, \quad (31)$$

$$(j = 2, 3, \dots, N),$$

式中, A' 表示 $A(T_2)$ 关于 T_2 求导, 各向量表达式为

$$\begin{cases} \mathbf{v}_{j1} = -2i\omega_j \mathbf{M}_j \mathbf{p}_j + \mathbf{P}_{12}(\mathbf{p}_j), \\ \mathbf{v}_{j2} = 4i\omega_j \mathbf{P}_{21}(\mathbf{p}_j, \mathbf{z}_{j1}) + 2i\omega_j (\mathbf{P}_{21}(\mathbf{z}_{j3}, \mathbf{p}_j) - \mathbf{P}_{21}(\mathbf{z}_{j1}, \mathbf{p}_j)) + 4\omega_j^2 \mathbf{M}_1(\mathbf{p}_j) \mathbf{z}_{j1} \\ + \omega_j^2 \mathbf{M}_1(\mathbf{z}_{j3}) \mathbf{p}_j + \omega_j^2 \mathbf{M}_1(\mathbf{z}_{j1}) \mathbf{p}_j \\ + 3\omega_j^2 \mathbf{M}_2(\mathbf{p}_j, \mathbf{p}_j) \mathbf{p}_j - \mathbf{K}_1(\mathbf{p}_j)(\mathbf{z}_{j1} + \mathbf{z}_{j3}) \\ - (\mathbf{K}_1(\mathbf{z}_{j1}) + \mathbf{K}_1(\mathbf{z}_{j3})) \mathbf{p}_j - 3\mathbf{K}_2(\mathbf{p}_j, \mathbf{p}_j) \mathbf{p}_j \\ - 2(\mathbf{K}_2(\mathbf{p}_j, \mathbf{z}_{j1}) + \mathbf{K}_2(\mathbf{p}_j, \mathbf{z}_{j3})) \boldsymbol{\eta}_0 \\ - 3\mathbf{K}_3(\mathbf{p}_j, \mathbf{p}_j, \mathbf{p}_j) \boldsymbol{\eta}_0, \\ \mathbf{v}_{jk} = \omega_j^2 \mathbf{M}_1(\mathbf{z}_{k3}) \mathbf{p}_1 + 2\omega_j^2 \mathbf{M}_2(\mathbf{p}_k, \mathbf{p}_k) \mathbf{p}_j \\ - \mathbf{K}_1(\mathbf{p}_j) \mathbf{z}_{k3} - \mathbf{K}_1(\mathbf{z}_{k3}) \mathbf{p}_j + 2i\omega_j \mathbf{P}_{21}(\mathbf{z}_{k3}, \mathbf{p}_j) \\ - 2(\mathbf{K}_2(\mathbf{p}_j, \mathbf{p}_k) \mathbf{p}_k + \mathbf{K}_2(\mathbf{p}_k, \mathbf{p}_j) \mathbf{p}_k \\ + \mathbf{K}_2(\mathbf{p}_k, \mathbf{p}_k) \mathbf{p}_j) - 6\mathbf{K}_3(\mathbf{p}_j, \mathbf{p}_k, \mathbf{p}_k) \boldsymbol{\eta}_0, \\ j = 1, 2, \dots, N, \end{cases} \quad (32)$$

其中,

$$\begin{cases} \mathbf{z}_{j1} = [\mathbf{R}_0 - 4\omega_j^2 \mathbf{M}_0]^{-1} [\omega_j^2 \mathbf{M}_1(\mathbf{p}_j) \mathbf{p}_j \\ - \mathbf{K}_2(\mathbf{p}_j, \mathbf{p}_j) \boldsymbol{\eta}_0 - \mathbf{K}_1(\mathbf{p}_j) \mathbf{p}_j \\ + i\omega_j \mathbf{P}_2(\mathbf{p}_j, \mathbf{p}_j)], \\ \mathbf{z}_{j3} = 2\mathbf{R}_0^{-1} [\omega_j^2 \mathbf{M}_1(\mathbf{p}_j) \mathbf{p}_j - \mathbf{K}_2(\mathbf{p}_j, \mathbf{p}_j) \boldsymbol{\eta}_0 \\ - \mathbf{K}_1(\mathbf{p}_j) \mathbf{p}_j], \\ \mathbf{z}_{k3} = 2\mathbf{R}_0^{-1} [\omega_k^2 \mathbf{M}_1(\mathbf{p}_k) \mathbf{p}_k - \mathbf{K}_2(\mathbf{p}_k, \mathbf{p}_k) \boldsymbol{\eta}_0 \\ - \mathbf{K}_1(\mathbf{p}_k) \mathbf{p}_k]. \end{cases} \quad (33)$$

由于 $\mathbf{u}_j$ 为特征向量, 可取 $\mathbf{u}_j$ 使得

$$\mathbf{u}_j^T \mathbf{v}_{j1} = 1,$$

这样方程(30)和(31)写为

$$A'_1 - 4i\alpha_1 A_1^2 \bar{A}_1 - 4i \sum_{k=2}^N A_1 A_k \bar{A}_k \alpha_{1k} + \frac{1}{2} i \alpha_{10} v_3 e^{i\omega T_2} = 0, \quad (34)$$

$$A'_j - 4i\alpha_j A_j^2 \bar{A}_j - 4i \sum_{k=1, k \neq j}^N A_j A_k \bar{A}_k \alpha_{jk} = 0, \quad (35)$$

$$(j = 2, 3, \dots, N),$$

其中,

$$\begin{cases} \alpha_1 = \frac{1}{4} i \mathbf{u}_1^T \mathbf{v}_{12}, \alpha_{1k} = \frac{1}{4} i \mathbf{u}_1^T \mathbf{v}_{1k}, \alpha_{10} = -2i v_0 \mathbf{u}_1^T \mathbf{L}, \\ \alpha_j = \frac{1}{4} i \mathbf{u}_j^T \mathbf{v}_{j2}, \alpha_{jk} = \frac{1}{4} i \mathbf{u}_j^T \mathbf{v}_{jk}, \end{cases} \quad (36)$$

设 $A_j(T_2) = \frac{1}{2} a_j(T_2) e^{i\beta_j(T_2)}$, 将 $\boldsymbol{\eta}_1, \boldsymbol{\eta}_2$ 代入 $\boldsymbol{\eta}$ 得

$$\begin{aligned} \boldsymbol{\eta}(t, \varepsilon) = & \boldsymbol{\eta}_0 + \varepsilon \left[\sum_{j=1}^N a_j(T_2) \cos(\omega_j T_0 + \beta_j(T)) \mathbf{p}_j \right] \\ & + \frac{1}{2} \varepsilon^2 \left\{ \sum_{r=1}^4 \sum_{s=1}^4 \Lambda(\omega_r + \omega_s) a_r(T_2) a_s(T_2) \right. \\ & \times \cos((\omega_r + \omega_s) T_0 + \beta_r(T_2) + \beta_s(T_2)) \mathbf{z}_{1rs} \\ & + \sum_{r=1}^4 \sum_{s=1}^4 \Lambda(\omega_r - \omega_s) a_r(T_2) a_s(T_2) \\ & \times \cos((\omega_r - \omega_s) T_0 + \beta_r(T_2) - \beta_s(T_2)) \mathbf{z}_{2rs} \left. \right\} + O(\varepsilon^3). \end{aligned} \quad (37)$$

假设 a 很小, 令 $\varepsilon a \rightarrow a$, 即 $\varepsilon \rightarrow 1$ 代入上式得

$$\begin{aligned} \boldsymbol{\eta}(t) = & \boldsymbol{\eta}_0 + a_1(t) \cos(\omega_1 t + \gamma(t)) \mathbf{p}_1 \\ & + \left[\sum_{j=2}^N a_j(t) \cos(\omega_j t + \beta_j(t)) \mathbf{p}_j \right] \\ & + \frac{1}{2} \left\{ \sum_{r=1}^4 \sum_{s=1}^4 \Lambda(\omega_r + \omega_s) a_r(t) a_s(t) \right. \\ & \times \cos((\omega_r + \omega_s) t + \beta_r(t) + \beta_s(t)) \mathbf{z}_{1rs} \\ & + \sum_{r=1}^4 \sum_{s=1}^4 \Lambda(\omega_r - \omega_s) a_r(t) a_s(t) \cos((\omega_r - \omega_s) t \\ & \left. + \beta_r(t) - \beta_s(t)) \mathbf{z}_{2rs} \right\} + O(a^3), \end{aligned} \quad (38)$$

其中,

$$\begin{cases} \gamma(t) = \beta_1(t) - \sigma t, \\ \mathbf{z}_{1rs} = \omega_s^2 \mathbf{M}_1(\mathbf{p}_r) \mathbf{p}_s - \omega_r \omega_s \mathbf{P}_{22}(\mathbf{p}_r, \mathbf{p}_s) \\ - \mathbf{K}_2(\mathbf{p}_r, \mathbf{p}_s) \boldsymbol{\eta}_0 - \mathbf{K}_1(\mathbf{p}_r) \mathbf{p}_s, \\ \mathbf{z}_{2rs} = \omega_s^2 \mathbf{M}_1(\mathbf{p}_r) \mathbf{p}_s + \omega_r \omega_s \mathbf{P}_{22}(\mathbf{p}_r, \mathbf{p}_s) \\ - \mathbf{K}_2(\mathbf{p}_r, \mathbf{p}_s) \boldsymbol{\eta}_0 - \mathbf{K}_1(\mathbf{p}_r) \mathbf{p}_s, \end{cases} \quad (39)$$

将

$$A_j(T_2) = \frac{1}{2} a_j(T_2) e^{i\beta_j(T_2)}$$

代入方程(34)和(35), 并分离实部和虚部, 则有

$$\begin{cases} \dot{a}_1 = -\alpha_{12}a_1^3 + \alpha_{102}v_3\cos\gamma - \alpha_{101}v_3\sin\gamma \\ \quad - \sum_{k=2}^N \alpha_{1k2}a_1a_k^2, \\ \dot{\gamma} = -\frac{\alpha_{102}v_3\sin\gamma + \alpha_{101}v_3\cos\gamma}{a_1} + \alpha_{11}a_1^2 \\ \quad + \sum_{k=2}^N \alpha_{1k1}a_k^2 - \sigma, \\ \dot{a}_j = -\alpha_{j2}a_j^3 - \sum_{k=1, k \neq j}^N \alpha_{jk2}a_ja_k^2, \\ \dot{\beta}_j = \alpha_{j1}a_j^2 + \sum_{k=1, k \neq j}^N \alpha_{jk1}a_j^2, (j=2, 3, \dots, N), \end{cases} \quad (40)$$

式中, $\alpha_{11}, \alpha_{101}, \alpha_{j1}, \alpha_{jk1}$ 和 $\alpha_{12}, \alpha_{102}, \alpha_{j2}, \alpha_{jk2}$ 分别为 $\alpha_1, \alpha_{10}, \alpha_j, \alpha_{jk}$ 的实部和虚部.

令方程(40)中 $\dot{a}_1 = 0, \dot{\gamma} = 0, \dot{a}_j = 0, \dot{\beta}_j = 0$ 得平衡点 $a_{1e}, \gamma_e, a_{je}, \beta_{je}$, 即

$$\begin{cases} -\alpha_{12}a_{1e}^3 + \alpha_{102}v_3\cos\gamma_e - \alpha_{101}v_3\sin\gamma_e - \sum_{k=2}^N \alpha_{1k2}a_{1e}a_{ke}^2 = 0, \\ \frac{\alpha_{102}v_3\sin\gamma_e + \alpha_{101}v_3\cos\gamma_e}{a_{1e}} - \alpha_{11}a_{1e}^2 - \sum_{k=2}^N \alpha_{1k1}a_{ke}^2 + \sigma = 0, \\ \alpha_{j2}a_{je}^3 + \sum_{k=1, k \neq j}^N \alpha_{jk2}a_{je}a_{ke}^2 = 0, \\ \alpha_{j1}a_{je}^2 + \sum_{k=1, k \neq j}^N \alpha_{jk1}a_{je}^2 = 0, (j=2, 3, \dots, N), \end{cases} \quad (41)$$

消去 γ 后获得频响方程如下:

$$a_{1e}^6\alpha_1^2 - 2a_{1e}^4\alpha_{11}\sigma + a_{1e}^2\sigma^2 - v_3^2\alpha_{10}^2 = 0. \quad (42)$$

该平衡点的稳定性依赖于方程(40)右端 Jacobian 矩阵的特征值.

4 算例分析

取结构参数为

$$m_1 = m_2 = 0.05 \text{ kg}, l_1 = l_2 = 0.1 \text{ m},$$

$$\rho = 0.0833 \text{ kg/m}^3, A = 1 \text{ m}^2, L = 0.3 \text{ m},$$

$$EI = 0.0908 \text{ N m}^2,$$

$$k_1 = k_2 = 0.92 \text{ N m/rad}, c = 1.$$

对系统第一阶模态的主共振响应进行数值分析.

图2给出了当 $v_0 = 0.1$ 时不同交流电压 v_3 作用下系统的主共振响应幅频曲线. 由图2可知, 对于足够小的交

流电压 v_3 , 振幅 a_1 是调谐参数 σ 的单值函数, 幅频响应曲线在非线性的共振点出现峰值. 随着 v_3 的增大, 曲线的峰向右倾斜, 呈现系统的硬特性特征. 由图3可知, 振幅和共振区随着交流电压 v_3 的增大而增大. 图4是在一个固定的激励频率下, 振幅 a_1 随着电压 v_3 的变化, 由图4可知振幅出现了跳跃现象.

图5给出了当 $v_0^2 = 1.65$ 时, 不同电压 v_3 作用下系统的主共振响应幅频曲线. 由图5可知, 此时随着 v_3 的增大, 曲线的共振峰向左倾斜, 系统呈现软特性特征, 振幅和共振区随交流电压的增大而增大.

对比图3和5可知, 系统呈现出软特性或硬特性特征与 v_0 有关, 这是由于 v_0 会影响系统的平衡状态. 图6给出了系统的软硬特性随 v_0 的增大而发生转变. 初始时

图2 幅频响应 ($v_0 = 0.1$)
Figure 2 Frequency-response curves at $v_0 = 0.1$.

图3 (网络版彩图) 不同 v_3 值对应的幅频响应 ($v_0 = 0.1$)
Figure 3 (Color online) Frequency-response curves of varying v_3 at $v_0 = 0.1$.

图4 模态幅值响应($\nu_0 = 0.1$, $\sigma = 1$)
Figure 4 Modal amplitude versus ν_3 at $\nu_0 = 0.1$, $\sigma = 1$.

图5 (网络版彩图)不同 ν_3 值对应的幅频响应曲线($\nu_0^2 = 1.65$)
Figure 5 (Color online) Frequency-response curves of varying ν_3 at $\nu_0^2 = 1.65$.

图6 非线性特性
Figure 6 The nonlinear characteristics of varying ν_0 .

系统只呈现出硬特性,随着 ν_0 的增大,在临界值处系统是局部线性的,幅频曲线不弯曲,继续增大 ν_0 系统的幅频曲线变为向左倾斜,系统的非线性转变为软特性特征。

5 结论

刚-柔耦合的大型空间天线结构动态响应具有多解,存在幅值跳跃等非线性现象.支撑天线的刚性机械臂在力矩电机的驱动激励下,一阶模态响应幅值随驱动电机的电压增加而增大,共振区也相应增大,同时导致模态幅值跳跃现象.驱动电机的直流电压很小时,系统表现为硬非线性特性,一旦直流电压超过临界值,则系统向软特性转变。

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附录

方程(6)中矩阵 $\mathbf{M}(\boldsymbol{\eta})$, $\mathbf{K}$ 及向量 $\mathbf{P}(\boldsymbol{\eta}, \dot{\boldsymbol{\eta}})$ 的表达式为

$$\mathbf{M}(\boldsymbol{\eta}) = \begin{bmatrix} M_{11} & M_{12} & M_{13} & M_{14} \\ M_{21} & M_{22} & M_{23} & M_{24} \\ M_{31} & M_{32} & M_{33} & M_{34} \\ M_{41} & M_{42} & M_{43} & M_{44} \end{bmatrix}, \quad (\text{a1})$$

其中,

$$\left\{ \begin{aligned} M_{11} &= J_1 + J_2 + \frac{1}{4}m_1l_1^2 + \frac{1}{4}m_2l_2^2 + m_2l_1^2 \\ &\quad + m_2l_1l_2\cos\theta_2 + \rho A\left[\left(\frac{2}{3}L^3 + 2L(l_1^2 + l_2^2)\right) \right. \\ &\quad \left. + 4Ll_1l_2\cos\theta_2 + 2A_xl_1(q_1 - q_2)\cos\theta_2 \right. \\ &\quad \left. + 2A_xl_2(q_1 - q_2) + A_{xx}(q_1^2 + q_2^2)\right], \\ M_{12} &= M_{21} = J_2 + \frac{1}{4}m_2l_2^2 + \frac{1}{2}m_2l_1l_2\cos\theta_2 \\ &\quad + \rho A\left[\left(\frac{2}{3}L^3 + 2Ll_2^2\right) \right. \\ &\quad \left. + 2Ll_1l_2\cos\theta_2 + A_xl_1(q_1 - q_2)\cos\theta_2 + 2A_xl_2(q_1 - q_2) \right. \\ &\quad \left. + A_{xx}(q_1^2 + q_2^2)\right], \\ M_{13} &= M_{31} = \rho A A_x l_1 \sin\theta_2 + \rho A B_x, \\ M_{14} &= M_{41} = -\rho A A_x l_1 \sin\theta_2 + \rho A B_x, \\ M_{22} &= J_2 + \frac{1}{4}m_2l_2^2 + \rho A\left[\left(\frac{2}{3}L^3 + 2Ll_2^2\right) \right. \\ &\quad \left. + 2A_xl_2(q_1 - q_2) + A_{xx}(q_1^2 + q_2^2)\right], \\ M_{23} &= M_{32} = M_{24} = M_{42} = \rho A B_x, \\ M_{33} &= M_{44} = \rho A A_{xx}, \\ M_{34} &= M_{43} = 0, \\ A_x &= \int_0^L \varphi(x)dx, A_{xx} = \int_0^L \varphi^2(x)dx, B_x \\ &= \int_0^L x\varphi(x)dx. \end{aligned} \right.$$

$$\mathbf{K} = \begin{bmatrix} k_1 & 0 & 0 & 0 \\ 0 & k_2 & 0 & 0 \\ 0 & 0 & c_8 & 0 \\ 0 & 0 & 0 & c_8 \end{bmatrix}, \quad (\text{a2})$$

其中,

$$c_8 = EI \int_0^L \left(\frac{d^2\varphi(x)}{dx^2} \right)^2 dx.$$

$$\mathbf{P}(\boldsymbol{\eta}, \dot{\boldsymbol{\eta}}) = [Q_{v1} \quad Q_{v2} \quad Q_{v3} \quad Q_{v4}]^T, \quad (\text{a3})$$

其中

$$\left\{ \begin{aligned} Q_{v1} &= (2\rho A L l_1 l_2 + \rho A A_x l_1 (q_1 - q_2)) \sin\theta_2 (\dot{\theta}_2^2 + 2\dot{\theta}_1 \dot{\theta}_2) \\ &\quad + m_2 l_1 l_2 \sin\theta_2 \dot{\theta}_1 \dot{\theta}_2 + 2\rho A [A_x l_1 \cos\theta_2 (-q_1 + q_2) \dot{\theta}_2 \\ &\quad + A_x (l_2 (\dot{\theta}_1 + \dot{\theta}_2) + l_1 \cos\theta_2 \dot{\theta}_1) (-\dot{q}_1 + \dot{q}_2) \\ &\quad - A_{xx} (q_1 \dot{q}_1 + q_2 \dot{q}_2) \dot{\theta}_2], \\ Q_{v2} &= -\frac{1}{2} m_2 l_1 l_2 \sin\theta_2 \dot{\theta}_1^2 + \rho A [A_x l_1 (-q_1 + q_2) \sin\theta_2 \dot{\theta}_1^2 \\ &\quad + 2A_x l_2 (-\dot{q}_1 + \dot{q}_2) (\dot{\theta}_1 + \dot{\theta}_2) - 2L l_1 l_2 \sin\theta_2 \dot{\theta}_1^2 \\ &\quad - 2A_{xx} (q_1 \dot{q}_1 + q_2 \dot{q}_2) (\dot{\theta}_1 + \dot{\theta}_2)], \\ Q_{v3} &= \rho A [A_x l_1 \cos\theta_2 \dot{\theta}_1^2 + (A_x l_2 + A_{xx} q_1) (\dot{\theta}_1 + \dot{\theta}_2)^2], \\ Q_{v4} &= -\rho A [A_x l_1 \cos\theta_2 \dot{\theta}_1^2 + (A_x l_2 - A_{xx} q_2) (\dot{\theta}_1 + \dot{\theta}_2)^2]. \end{aligned} \right.$$

Nonlinear primary resonance of a rigid-flexible space symmetric antenna

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This paper studies the nonlinear response of a T-shape space antenna structure composed of two rigid arms jointed by a spring and two uniform beams made by isotropic material. Based on the Lagrange equation and the assumed modes method, the nonlinear dynamic equations of in-plane motion of the rigid-flexible coupled antenna are established, with four degrees of freedom. In order to reveal the dynamic characteristics of the nonlinear system, the method of multiple scales expressed in matrix form is used to solve the primary resonance responses of third-order approximation. Analyzing the frequency-amplitude response gives the jumping phenomenon in the primary resonance response. With increase of the excitation amplitude, the frequency-amplitude response curve would have a conversion between the hardening and softening types. Under small excitation amplitude, the frequency-response curve exhibits the hardening-type behavior, whereas the softening-type behavior would appear once the excitation amplitude is out to a critical value. In the case of softening-type behavior, the resonance frequencies decrease with increase of the excitation amplitudes.

rigid-flexible coupled structures, space antenna, nonlinear resonance, four-degree-of-freedom, assumed modes method

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