不可微多目标规划的高阶对称对偶性

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收稿日期: 2011-11-22; 接受日期: 2013-02-21; * 通信作者

国家自然科学基金 (批准号: 10831009 和 11271391) 和重庆市自然科学基金 (批准号: CSTC2011BA0030) 资助项目

摘要 本文研究锥约束不可微多目标规划的 Mond-Weir 型高阶对称对偶问题. 本文指出 Agarwal 等人 (2010) 和 Gupta 等人 (2010) 工作的不足, 给出规划问题的强对偶和逆对偶定理.

关键词 多目标优化 高阶对称对偶 锥约束 强对偶定理

MSC (2010) 主题分类 90C29, 90C46

1 引言

设 $C_1 \subseteq \mathbb{R}^n$ 和 $C_2 \subseteq \mathbb{R}^m$ 为闭凸锥, $S_1 \subseteq \mathbb{R}^n$ 和 $S_2 \subseteq \mathbb{R}^m$ 是满足 $C_1 \times C_2 \subseteq S_1 \times S_2$ 的开集.

Chen^[1] 研究了不可微多目标规划的高阶对称对偶问题, 并在高阶 F -凸性假设下给出了对偶定理. Agarwal 等人^[2] 和 Gupta 等人^[3] 将文献 [1] 中的自然锥序推广到了一般的锥序, 并分别在高阶 $K-F$ -凸性和高阶 (F, α, ρ, d) -凸性假设下给出了相应的对偶定理.

本文考虑如下不可微多目标高阶对称对偶问题:

$$\begin{aligned}
 (\text{MP}) \quad & K\text{-minimize } (f_1(x, y) + s(x \mid D_1) - y^T z_1 + h_1(x, y, p_1) - p_1[\nabla_{p_1} h_1(x, y, p_1)], \dots, \\
 & f_k(x, y) + s(x \mid D_k) - y^T z_k + h_k(x, y, p_k) - p_k[\nabla_{p_k} h_k(x, y, p_k)]) \\
 \text{s.t.} \quad & - \sum_{i=1}^k \lambda_i (\nabla_y f_i(x, y) - z_i + \nabla_{p_i} h_i(x, y, p_i)) \in C_2^*, \quad (1.1a)
 \end{aligned}$$

$$y^T \sum_{i=1}^k \lambda_i (\nabla_y f_i(x, y) - z_i + \nabla_{p_i} h_i(x, y, p_i)) \geq 0, \quad (1.1b)$$

$$\lambda \in \text{int} K^*, \quad x \in C_1, \quad (1.1c)$$

$$z_i \in E_i, \quad i = 1, 2, \dots, k, \quad (1.1d)$$

$$\begin{aligned}
 (\text{MD}) \quad & K\text{-maximize } (f_1(u, v) - s(v \mid E_1) + u^T \omega_1 + g_1(u, v, r_1) - r_1[\nabla_{r_1} g_1(u, v, r_1)], \dots, \\
 & f_k(u, v) - s(v \mid E_k) + u^T \omega_k + g_k(u, v, r_k) - r_k[\nabla_{r_k} g_k(u, v, r_k)]) \\
 \text{s.t.} \quad & \sum_{i=1}^k \lambda_i (\nabla_x f_i(u, v) + \omega_i + \nabla_{r_i} g_i(u, v, r_i)) \in C_1^*, \quad (1.2a)
 \end{aligned}$$

英文引用格式: Yang X M, Yang J. Higher order symmetric duality in nondifferentiable multiobjective programming problems involving cones (in Chinese). Sci Sin Math, 2013, 43: 703-708, doi: 10.1360/012011-914

$$u^T \sum_{i=1}^k \lambda_i (\nabla_x f_i(u, v) + \omega_i + \nabla_{r_i} f_i(u, v, r_i)) \leq 0, \quad (1.2b)$$

$$\lambda \in \text{int} K^*, \quad v \in C_2, \quad (1.2c)$$

$$\omega_i \in D_i, \quad i = 1, 2, \dots, k, \quad (1.2d)$$

其中, D_i 和 E_i , $i = 1, 2, \dots, k$ 分别为 $\mathbb{R}^n$ 和 $\mathbb{R}^m$ 中的紧凸集; $f_i: S_1 \times S_2 \rightarrow \mathbb{R}$, $h_i: S_1 \times S_2 \times \mathbb{R}^m \rightarrow \mathbb{R}$ 和 $g_i: S_1 \times S_2 \times \mathbb{R}^n \rightarrow \mathbb{R}$ 是可微函数, $i = 1, \dots, k$. C_1^* 和 C_2^* 分别为 C_1 和 C_2 的正极锥, $\lambda = (\lambda_1, \lambda_2, \dots, \lambda_k)^T \in \mathbb{R}^k$.

Agarwal 等人^[2] 在高阶 F -凸性假设下证明了弱对偶定理, 并得到了如下强对偶定理.

定理 1.1 (强对偶定理) 设 $(\bar{x}, \bar{y}, \bar{\lambda}, \bar{z}, \bar{z}_1, \bar{z}_2, \dots, \bar{z}_k, \bar{p}_1, \bar{p}_2, \dots, \bar{p}_k)$ 为问题 (MP) 的真有效解, $f_i: S_1 \times S_2 \rightarrow \mathbb{R}$ 在点 $(\bar{x}, \bar{y})$ 处三阶可导, $h_i: S_1 \times S_2 \times \mathbb{R}^m \rightarrow \mathbb{R}$ 在点 $(\bar{x}, \bar{y}, \bar{p}_i)$ 处二阶可导, $g_i: S_1 \times S_2 \times \mathbb{R}^n \rightarrow \mathbb{R}$ 可微, $i = 1, 2, \dots, k$. 若以下条件成立:

(I) $h_i(\bar{x}, \bar{y}, 0) = 0$, $g_i(\bar{x}, \bar{y}, 0) = 0$, $\nabla_{p_i} h_i(\bar{x}, \bar{y}, 0) = 0$, $\nabla_y h_i(\bar{x}, \bar{y}, 0) = 0$, $\nabla_x h_i(\bar{x}, \bar{y}, 0) = \nabla_{r_i} g(\bar{x}, \bar{y}, 0)$, $i = 1, 2, \dots, k$;

(II) 对所有的 $i \in \{1, 2, \dots, k\}$, Hesse 矩阵 $\nabla_{p_i p_i} h_i(\bar{x}, \bar{y}, \bar{p}_i)$ 正定或负定;

(III) 向量组 $\{\nabla_y f_i(\bar{x}, \bar{y}) - \bar{z}_i + \nabla_{p_i} h_i(\bar{x}, \bar{y}, \bar{p}_i)\}_{i=1}^k$ 线性无关;

(IV) 向量组 $\{\nabla_y f_i(\bar{x}, \bar{y}) - \bar{z}_i + \nabla_{p_i} h_i(\bar{x}, \bar{y}, \bar{p}_i), \nabla_y f_i(\bar{x}, \bar{y}) - \bar{z}_i + \nabla_{p_i} h_i(\bar{x}, \bar{y}, \bar{p}_i)\}_{i=1}^k$ 线性无关;

(V) 对某 $\alpha \in \text{int} K^*$ 和 $\bar{p}_i \in \mathbb{R}^n$, $\bar{p}_i \neq 0$ ($i = 1, 2, \dots, k$) 意味着

$$\sum_{i=1}^k \alpha_i \bar{p}_i^T [\nabla_y f_i(\bar{x}, \bar{y}) - \bar{z}_i + \nabla_{p_i} h_i(\bar{x}, \bar{y}, \bar{p}_i)] \neq 0;$$

(VI) K 是满足条件 $\mathbb{R}_+^k \subseteq K$ 的闭凸锥, 则有

(i) $\bar{p}_i = 0, i = 1, 2, \dots, k$;

(ii) 存在 $\bar{w}_i \in D_i, i = 1, 2, \dots, k$ 使得 $(\bar{x}, \bar{y}, \bar{\lambda}, \bar{w}_1, \bar{w}_2, \dots, \bar{w}_k, \bar{r}_1 = \bar{r}_2 = \dots = \bar{r}_k = 0)$ 为问题 (MD) 的可行解. 此外, 若文献 [2, 定理 3.1] 中的条件成立, 则 $(\bar{x}, \bar{y}, \bar{\lambda}, \bar{w}_1, \bar{w}_2, \dots, \bar{w}_k, \bar{r}_1 = \bar{r}_2 = \dots = \bar{r}_k = 0)$ 为问题 (MD) 的真有效解, 且原问题和对偶问题有相同的最优值.

文献 [2] 中的强对偶定理存在以下的错误:

(a) 定理 1.1(IV) 中 α 与定理证明中获得 α 一般来讲是不相同的. 因此, 此定理的证明是错误的;

(b) 基于定理 1.1(I), 我们有

$$\nabla_{p_i} h_i(\bar{x}, \bar{y}, 0) = 0, \quad \nabla_y h_i(\bar{x}, \bar{y}, 0) = 0, \quad i = 1, 2, \dots, k.$$

然而, 定理 1.1(i) 又为

$$\bar{p}_i = 0, \quad i = 1, 2, \dots, k.$$

显然, 向量组 $\{\nabla_y f_i(\bar{x}, \bar{y}) - \bar{z}_i + \nabla_{p_i} h_i(\bar{x}, \bar{y}, \bar{p}_i), \nabla_y f_i(\bar{x}, \bar{y}) - \bar{z}_i + \nabla_{p_i} h_i(\bar{x}, \bar{y}, \bar{p}_i), i = 1, 2, \dots, k\}$ 不是线性无关的. 这表明, 定理中的条件 (IV) 是错误的;

(c) 在上面的强对偶定理中, 条件 (IV) 蕴含着条件 (III).

事实上, 在文献 [2, 定理 3.3], [4, 定理 3.3, 3.4 和 4.1] 和 [3, 定理 4.3 和 4.4] 都存在同样的错误.

2 强对偶定理

本节修正定理 1.1 中的错误, 给出问题 (MP) 和 (MD) 正确的强对偶定理.

定理 2.1 (强对偶定理) 设 $(\bar{x}, \bar{y}, \bar{\lambda}, \bar{z}_1, \bar{z}_2, \dots, \bar{z}_k, \bar{p}_1, \bar{p}_2, \dots, \bar{p}_k)$ 为问题 (MP) 的真有效解. 假设 $f_i: S_1 \times S_2 \rightarrow \mathbb{R}$ 在点 $(\bar{x}, \bar{y})$ 处有三阶可导, $h_i: S_1 \times S_2 \times \mathbb{R}^m \rightarrow \mathbb{R}$ 在点 $(\bar{x}, \bar{y}, \bar{p}_i)$ 处二阶可导, $g_i: S_1 \times S_2 \times \mathbb{R}^n \rightarrow \mathbb{R}$ 可微, $i = 1, 2, \dots, k$. 若以下条件成立:

(I) $h_i(\bar{x}, \bar{y}, 0) = 0, g_i(\bar{x}, \bar{y}, 0) = 0, \nabla_{p_i} h_i(\bar{x}, \bar{y}, 0) = 0, \nabla_y h_i(\bar{x}, \bar{y}, 0) = 0, \nabla_x h_i(\bar{x}, \bar{y}, 0) = \nabla_{r_i} g(\bar{x}, \bar{y}, 0),$
 $i = 1, 2, \dots, k$;

(II) 对每个 $i = 1, 2, \dots, k$, Hesse 矩阵 $\nabla_{p_i p_i} h_i(\bar{x}, \bar{y}, \bar{p}_i)$ 正定或负定;

(III) 向量组 $\{\nabla_y f_i(\bar{x}, \bar{y}) - \bar{z}_i + \nabla_{p_i} h_i(\bar{x}, \bar{y}, \bar{p}_i)\}_{i=1}^k$ 线性无关;

(IV) 存在函数 $\bar{h}$ 和 $\bar{\bar{h}}$ 满足

$$\nabla_y h_i(\bar{x}, \bar{y}, \bar{p}_i) = \bar{h}_i(\bar{x}, \bar{y}, \bar{p}_i) \bar{p}_i, \quad \nabla_{p_i} h_i(\bar{x}, \bar{y}, \bar{p}_i) = \bar{\bar{h}}_i(\bar{x}, \bar{y}, \bar{p}_i) \bar{p}_i;$$

(V) $\bar{p}_i \in \mathbb{R}^n, (\bar{p}_1, \bar{p}_2, \dots, \bar{p}_k) \neq 0$ 蕴含着

$$\sum_{i=1}^k \bar{\lambda}_i \bar{p}_i^T [\nabla_y f_i(\bar{x}, \bar{y}) - \bar{z}_i + \nabla_{p_i} h_i(\bar{x}, \bar{y}, \bar{p}_i)] \neq 0;$$

(VI) K 为满足条件 $\mathbb{R}_+^k \subseteq K$ 的闭凸锥,

则有

(i) $\bar{p}_i = 0, i = 1, 2, \dots, k$;

(ii) 存在 $\bar{w}_i \in C_i, i = 1, 2, \dots, k$ 使得 $(\bar{x}, \bar{y}, \bar{\lambda}, \bar{w}_1, \bar{w}_2, \dots, \bar{w}_k, \bar{r}_1 = \bar{r}_2 = \dots = \bar{r}_k = 0)$ 为问题 (MD) 的可行解. 此外, 若文献 [2, 定理 3.1] 中的条件成立, 则 $(\bar{x}, \bar{y}, \bar{\lambda}, \bar{w}_1, \bar{w}_2, \dots, \bar{w}_k, \bar{r}_1 = \bar{r}_2 = \dots = \bar{r}_k = 0)$ 为问题 (MD) 的真有效解, 且原问题和对偶问题有相同的最优值.

证明 因 $(\bar{x}, \bar{y}, \bar{\lambda}, \bar{z}_1, \bar{z}_2, \dots, \bar{z}_k, \bar{p}_1, \bar{p}_2, \dots, \bar{p}_k)$ 是问题 (MP) 的有效解, 故由文献 [5] 中的 Fritz John 型最优性条件可知, 存在 $\alpha \in K^*, \beta \in C_2, \mu \in \mathbb{R}_+$ 和 $\xi_i \in D_i, i = 1, 2, \dots, k$, 使得

$$\left[\sum_{i=1}^k \alpha_i [\nabla_x f_i(\bar{x}, \bar{y}) + \xi_i + \nabla_x h_i(\bar{x}, \bar{y}, \bar{p}_i)] + \sum_{i=1}^k \bar{\lambda}_i (\nabla_{yx} f_i(\bar{x}, \bar{y}))^T (\beta - \mu \bar{y}) \right. \\ \left. + \sum_{i=1}^k (\nabla_{p_i x} h_i(\bar{x}, \bar{y}, \bar{p}_i))^T (\lambda_i \beta - \alpha_i \bar{p}_i - \bar{\lambda}_i \mu \bar{y}) \right] (x - \bar{x}) \geq 0, \quad \forall x \in C_1, \quad (2.1)$$

$$\sum_{i=1}^k \alpha_i [\nabla_y f_i(\bar{x}, \bar{y}) - \bar{z}_i + \nabla_y h_i(\bar{x}, \bar{y}, \bar{p}_i)] + \sum_{i=1}^k \bar{\lambda}_i (\nabla_{yy} f_i(\bar{x}, \bar{y}))^T (\beta - \mu \bar{y}) \\ + \sum_{i=1}^k (\nabla_{p_i y} h_i(\bar{x}, \bar{y}, \bar{p}_i))^T (\bar{\lambda}_i \beta - \alpha_i \bar{p}_i - \bar{\lambda}_i \mu \bar{y}) - \mu \sum_{i=1}^k \bar{\lambda}_i [\nabla_y f_i(\bar{x}, \bar{y}) - \bar{z}_i + \nabla_{p_i} h_i(\bar{x}, \bar{y}, \bar{p}_i)] = 0, \quad (2.2)$$

$$(\nabla_{p_i p_i} h_i(\bar{x}, \bar{y}, \bar{p}_i))^T (\bar{\lambda}_i \beta - \alpha_i \bar{p}_i - \bar{\lambda}_i \mu \bar{y}) = 0, \quad i = 1, 2, \dots, k, \quad (2.3)$$

$$(\beta - \mu \bar{y})^T [\nabla_y f_i(\bar{x}, \bar{y}) - \bar{z}_i + \nabla_{p_i} h_i(\bar{x}, \bar{y}, \bar{p}_i)] (\lambda - \bar{\lambda}) \geq 0, \quad \forall \lambda \in \text{int} K^*, \quad i = 1, 2, \dots, k, \quad (2.4)$$

$$\beta^T \sum_{i=1}^k \bar{\lambda}_i [\nabla_y f_i(\bar{x}, \bar{y}) - \bar{z}_i + \nabla_{p_i} h_i(\bar{x}, \bar{y}, \bar{p}_i)] = 0, \quad (2.5)$$

$$\mu \bar{y}^T \sum_{i=1}^k \bar{\lambda}_i [\nabla_y f_i(\bar{x}, \bar{y}) - \bar{z}_i + \nabla_{p_i} h_i(\bar{x}, \bar{y}, \bar{p}_i)] = 0, \quad (2.6)$$

$$\alpha_i \bar{y} + \bar{\lambda}_i \beta - \bar{\lambda}_i \mu \bar{y} \in N_{E_i}(\bar{z}_i), \quad i = 1, 2, \dots, k, \quad (2.7)$$

$$\xi_i^T \bar{x} = s(\bar{x} \mid D_i), \quad i = 1, 2, \dots, k, \quad (2.8)$$

$$(\alpha, \beta, \mu) \neq 0. \quad (2.9)$$

不等式 (2.4) 等价于

$$(\beta - \mu \bar{y})^T [\nabla_y f_i(\bar{x}, \bar{y}) - \bar{z}_i + \nabla_{p_i} h_i(\bar{x}, \bar{y}, \bar{p}_i)] = 0, \quad i = 1, 2, \dots, k. \quad (2.10)$$

由假设 (II) 和 (2.3) 有

$$(\beta - \mu \bar{y}) \bar{\lambda}_i = \alpha_i \bar{p}_i, \quad i = 1, 2, \dots, k. \quad (2.11)$$

下面证明, 对所有的 $i \in \{1, \dots, k\}$, $\alpha_i \neq 0$. 假设对某个 j 有 $\alpha_j = 0$. 因 $\mathbb{R}_+^k \subseteq K$ 蕴含着 $K^* \subseteq \mathbb{R}_+^k$, 故 $\bar{\lambda} > 0$. 从而由 (2.11) 得

$$\beta = \mu \bar{y}. \quad (2.12)$$

将 (2.11) 和 (2.12) 代入 (2.2) 得

$$\sum_{i=1}^k \{ \alpha_i (\nabla_y f_i(\bar{x}, \bar{y}) - \bar{z}_i + \nabla_{p_i} h_i(\bar{x}, \bar{y}, \bar{p}_i)) - \mu \bar{\lambda}_i [\nabla_y f_i(\bar{x}, \bar{y}) - \bar{z}_i + \nabla_{p_i} h_i(\bar{x}, \bar{y}, \bar{p}_i)] \} = 0. \quad (2.13)$$

由假设 (IV) 和 (2.13) 有

$$\sum_{i=1}^k (\alpha_i - \mu \bar{\lambda}_i) [\nabla_y f_i(\bar{x}, \bar{y}) - \bar{z}_i + \nabla_{p_i} h_i(\bar{x}, \bar{y}, \bar{p}_i)] = 0.$$

再由假设 (III) 得

$$\alpha_i = \mu \bar{\lambda}_i, \quad i = 1, 2, \dots, k. \quad (2.14)$$

由 $\bar{\lambda}_i > 0$, $i = 1, 2, \dots, k$ 和对某个 j , $\alpha_j = 0$ 可知 $\mu = 0$. 因此, 由 (2.12), (2.14) 和 $\mu = 0$ 得 $\beta = 0$, $\alpha_i = 0$, $i = 1, 2, \dots, k$. 这与 $(\alpha, \beta, \mu) \neq 0$ 矛盾. 从而, 对所有的 $i \in \{1, \dots, k\}$, $\alpha_i \neq 0$. 又因 $\alpha \in K^* \subseteq \mathbb{R}_+^k$, 故

$$\alpha_i > 0, \quad i = 1, 2, \dots, k.$$

将 (2.11), $\alpha > 0$ 和 $\bar{\lambda} > 0$ 代入 (2.10) 得

$$\bar{p}_i^T [\nabla_y f_i(\bar{x}, \bar{y}) - \bar{z}_i + \nabla_{p_i} h_i(\bar{x}, \bar{y}, \bar{p}_i)] = 0, \quad i = 1, 2, \dots, k,$$

这表明,

$$\sum_{i=1}^k \bar{\lambda}_i \bar{p}_i^T [\nabla_y f_i(\bar{x}, \bar{y}) - \bar{z}_i + \nabla_{p_i} h_i(\bar{x}, \bar{y}, \bar{p}_i)] = 0.$$

由假设 (V) 可得

$$\bar{p}_i = 0, \quad i = 1, 2, \dots, k.$$

余下的证明可由文献 [2, 定理 3.2] 中的证明得到. □

同理, 我们对文献 [2, 定理 3.3] 进行了如下修正.

定理 2.2 (逆对偶定理) 设 $(\bar{u}, \bar{v}, \bar{\lambda}, \bar{w}_1, \bar{w}_2, \dots, \bar{w}_k, \bar{r}_1, \bar{w}_2, \dots, \bar{r}_k)$ 为问题 (MD) 的真有效解, 假设 $f_i : S_1 \times S_2 \rightarrow \mathbb{R}$ 在点 $(\bar{u}, \bar{v})$ 处三阶可导, $h_i : S_1 \times S_2 \times \mathbb{R}^m \rightarrow \mathbb{R}$ 在点 $(\bar{u}, \bar{v}, \bar{r}_i)$ 处二阶可导, $g_i : S_1 \times S_2 \times \mathbb{R}^n \rightarrow \mathbb{R}$ 可微, $i = 1, 2, \dots, k$. 若如下条件成立:

- (I) $h_i(\bar{u}, \bar{v}, 0) = 0, g_i(\bar{u}, \bar{v}, 0) = 0, \nabla_{r_i} h_i(\bar{u}, \bar{v}, 0) = 0, \nabla_x h_i(\bar{u}, \bar{v}, 0) = 0, \nabla_y h_i(\bar{u}, \bar{v}, 0) = \nabla_{p_i} g(\bar{u}, \bar{v}, 0),$
 $i = 1, 2, \dots, k;$
 (II) 对每个 $i = 1, 2, \dots, k$, Hesse 矩阵 $\nabla_{r_i r_i} h_i(\bar{u}, \bar{v}, \bar{r}_i)$ 正定或负定;
 (III) 向量组 $\{\nabla_x f_i(\bar{u}, \bar{v}) + \bar{w}_i + \nabla_{r_i} h_i(\bar{u}, \bar{v}, \bar{r}_i)\}_{i=1}^k$ 线性无关;
 (IV) 存在函数 $\bar{h}$ 和 $\bar{\bar{h}}$ 满足

$$\nabla_y h_i(\bar{u}, \bar{v}, \bar{r}_i) = \bar{h}_i(\bar{u}, \bar{v}, \bar{r}_i) \bar{r}_i, \quad \nabla_{\bar{r}_i} h_i(\bar{u}, \bar{v}, \bar{r}_i) = \bar{\bar{h}}_i(\bar{u}, \bar{v}, \bar{r}_i) \bar{r}_i;$$

- (V) $\bar{r}_i \in \mathbb{R}^n, (\bar{r}_1, \bar{r}_2, \dots, \bar{r}_k) \neq 0$ 蕴含着

$$\sum_{i=1}^k \bar{\lambda}_i \bar{r}_i^T [\nabla_x f_i(\bar{u}, \bar{v}) + \bar{w}_i + \nabla_{r_i} h_i(\bar{u}, \bar{v}, \bar{r}_i)] \neq 0;$$

- (VI) K 为满足条件 $\mathbb{R}_+^k \subseteq K$ 的闭凸锥,
 则有

- (i) $\bar{r}_i = 0, i = 1, 2, \dots, k;$
 (ii) 存在 $\bar{z}_i \in E_i, i = 1, 2, \dots, k$ 使得 $(\bar{u}, \bar{v}, \bar{\lambda}, \bar{z}_1, \bar{z}_2, \dots, \bar{z}_k, \bar{p}_1 = \bar{p}_2 = \dots = \bar{p}_k = 0)$ 为问题 (MP) 的可行解. 此外, 若文献 [2, 定理 3.1] 中的条件成立, 则 $(\bar{u}, \bar{v}, \bar{\lambda}, \bar{z}_1, \bar{z}_2, \dots, \bar{z}_k, \bar{p}_1 = \bar{p}_2 = \dots = \bar{p}_k = 0)$ 为问题 (MP) 的真有效解, 且原问题和对偶问题有相同的最优值.

注 2.1 (i) 定理 2.1 和 2.2 修正了文献 [2, 定理 3.2 和 3.3] 中的错误. 若 $D_i = \{0\}, E_i = \{0\}, i = 1, 2, \dots, k$, 则问题 (MP) 和 (MD) 退化为文献 [4] 中考虑的问题. 而定理 2.1 和 2.2 也对文献 [4, 定理 3.3, 3.4 和 4.1] 进行了修正. 此外, 若 $K = \mathbb{R}_+^m$, 定理 2.1 和 2.2 也修正了文献 [3, 定理 4.3 和 4.4].

- (ii) 若

$$h_i(x, y, p) = \frac{1}{2} p_i^T \nabla_{yy} f_i(x, y) p_i, \quad g_i(u, v, r) = \frac{1}{2} r_i^T \nabla_{xx} f_i(u, v) r_i,$$

则问题 (MP) 和 (MD) 退化为文献 [6] 中考虑的二阶对偶模型. 而定理 2.1 和 2.2 可以退化为文献 [6, 定理 3.4 和 3.5]. 此外, 若 $K = \mathbb{R}_+^n, C_1 = \mathbb{R}^n, C_2 = \mathbb{R}_+^m$, 问题 (MP) 和 (MD) 退化为文献 [7] 中考虑的对偶问题, 当把定理 2.1 和 2.2 中的有真有效解换为弱有效解时, 定理 2.1 和 2.2 中的结论就退化为文献 [7, 定理 2.1 和 2.2] 相应的结论.

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Higher order symmetric duality in nondifferentiable multiobjective programming problems involving cones

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Abstract In this paper, strong duality and converse duality theorems are established for a pair of Mond-Weir type multiobjective higher order symmetric dual programs over arbitrary cones. Our results fill some gaps in works of Agarwal et al. (2010) and Gupta et al. (2010).

Keywords multiobjective programming, higher order symmetric dual model, cone constraints, strong duality theorem

MSC(2010) 90C29, 90C46

doi: 10.1360/012011-914