联合噪声信道下基于部分纠缠信道的非定域 Toffoli 门远程实现

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摘要 量子操控远程实现是量子通信重要任务之一, 最近, 基于量子纠缠信道的非定域量子门引起了广泛关注. 基于不同信道提出了双量子比特控制非门, 三量子比特 Toffoli 门远程实现方案. 本文研究信道联合噪声下基于部分纠缠信道的三粒子非定域 Toffoli 门远程实现方案, 提出了两个非最大纠缠信道下可克制信道联合噪声的非定域 Toffoli 门远程实现方案. 一个方案用于联合退相位噪声下基于部分纠缠信道的非定域 Toffoli 门远程实现, 另一个用于联合转动噪声下非定域 Toffoli 门远程实现. 通信方先使用退相干无关子空间克制信道联合噪声影响, 再通过引入附加粒子和执行联合幺正演化消除部分纠缠信道对非定域 Toffoli 门远程实现影响. 通讯方仅需使用部分纠缠态来实现非定域量子门, 与其他方案相比, 具有可行性强的优点.

关键词 非定域 Toffoli 门, 联合退相位噪声, 联合转动噪声, 部分纠缠态

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1 引言

量子通信以量子态为信息载体, 网络中不同节点间量子态的远程传送与量子门远程实现是量子通信的重要任务之一^[1-5]. 基于预先共享量子纠缠态非定域相关性, 量子通信不仅可实现任意已知或未知量子态的远程传送^[6-24], 也可以实现任意量子门的远程传输^[25-32].

自从1995年Barenco等人^[33-36]证明任意普适量子门都可由单量子比特门和双量子比特控制非门(Con-

trolled-NOT, CNOT)或由单量子比特Hadamard门和三量子比特Toffoli门构建以来, 任意单量子比特门, CNOT门和Toffoli门远程实现引起了研究者广泛关注. 一方面, 不同研究组以纠缠光子为信道完成了任意单光子旋转操作^[37]、双光子非定域控制非门操作^[38]. 另一方面, 基于不同量子系统人们开展不同量子门, 特别是非定域量子门远程实现方案研究. 例如: Eisert等人^[39]建立了非定域CNOT门远程实现方案, 基于预先共享双粒子纠缠信道, 通讯方传递2比特经典信息可实现不同节点之间的非定域CNOT门操作. Huelga等

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人^[40]以Bell态为纠缠信道提出了任意单量子比特操控和某些特殊单量子比特操控远程实现方案. 2013年, Wang等人^[29]提出了基于固态系统的双量子非定域CNOT门模型. 2015年, Chen和Lu^[41]提出了基于Greenberger-Horne-Zeilinger (GHZ)态的多目标非定域Toffoli门模型. 2016年, Chen和Lu^[42]进一步将Eisert等人的非定域量子门方案推广到任意高维, 以三维Bell态 $|\varphi\rangle = \frac{1}{\sqrt{3}}(|00\rangle + |11\rangle + |22\rangle)$ 实现了高维非定域Toffoli门操作. 2018年, 提出了基于非对称GHZ态的两目标位、N控制位非定域量子门方案^[43]. 到目前为止, 已经完成了两个逻辑比特之间确定的非定域CNOT门实验^[44].

在量子通信网络中, 光量子态由于传输距离远、速度快等优点, 是远程量子通信的理想信息载体. 然而光子在传输过程中容易受到信道噪声影响. 噪声会导致加载在光子上的量子信息失真, 影响通信效率和通信安全. 为克制信道噪声影响, 采用的方法有纠缠纯化^[45-47]、量子避错^[48]以及量子消相干无关子空间(Decoherence Free Subspace, DFS)等^[49,50]. 在使用消相干无关子空间的量子信息处理方案中, 用多个量子比特编码一个逻辑比特, 不同量子比特所受信道噪声影响互相抵消, 从而整体表现为不受信道噪声影响. 近年来, 基于消相干无关子空间的联合噪声信道下的量子信息处理引起研究者广泛关注^[51-56]. Yamamoto等人^[51]提出了基于联合噪声信道的光量子态分配方案. Li等人^[52]提出了联合噪声信道下可克制信道联合噪声的量子密钥分配方案. 2010年, Dong等人^[53]提出了联合噪声信道下基于6光子纠缠态的量子可控离物传态协议. 顾斌等人^[54]建立了联合噪声信道下的量子安全直接通信模型. 2012年, 提出了联合噪声信道下任意多粒子态可离物传态和量子态联合制备模型^[55,56]. 2016年, Takeuchi等人^[57]开展了联合噪声信道下基于退相干无关子空间的盲量子计算研究. 2018年提出了联合噪声信道下基于线性光学元件的盲量子计算方案^[58].

虽然已有一些基于联合噪声信道的量子通信方案, 但是大多数方案都是联合噪声环境下量子态远程传送研究^[55,56]. 基于联合噪声环境的量子门远程传输研究得还不多. 本文提出两个联合噪声环境下非定域Toffoli门远程实现方案, 一个方案用于信道联合退相

位噪声下基于非最大纠缠信道的非定域Toffoli门远程实现, 另一个用于信道联合转动噪声下的非定域Toffoli门远程实现. 方案先用退相干无关子空间克制信道联合噪声影响, 再通过引入附加粒子和执行联合么正演化来消除非最大纠缠信道影响, 方案成功率与信道纠缠度相等. 由于通讯方仅需使用非最大纠缠信道就可完成非定域量子门操作, 与其他方案相比, 具有可行性强的优点.

2 联合退相位噪声下基于部分纠缠态的非定域Toffoli门

信道联合退相位噪声可表示为^[55,56]

$$U_c|0\rangle = |0\rangle, U_c|1\rangle = e^{i\theta}|1\rangle, \quad (1)$$

其中, θ 为联合噪声参数. $|0\rangle, |1\rangle$ 为测量基 σ_z 的两个本征态. 另一组正交测量基 σ_x 可表示为

$$|0\rangle_x = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle), |1\rangle_x = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle). \quad (2)$$

单量子比特操作 $\mathbf{X}, \mathbf{Z}$ 可实现不同量子态间的转化:

$$\mathbf{X} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \mathbf{Z} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \quad (3)$$

与文献^[55]类似, $\mathbf{X}^0=\mathbf{I}, \mathbf{X}^1=\mathbf{X}, \mathbf{Z}^0=\mathbf{I}, \mathbf{Z}^1=\mathbf{Z}$. 其中 $\mathbf{I}$ 为单位矩阵.

以第一个量子比特为控制位, 第二个量子比特为目标位, 2量子比特CNOT门可表示为^[59]

$$U_{\text{CNOT}} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}. \quad (4)$$

控制非门可将双量子比特态 $|j_1 j_2\rangle$ 转化为相应量子态 $(j_1, j_2=0, 1)$:

$$U_{\text{CNOT}}|j_1 j_2\rangle = |j_1, j_2 \oplus j_1\rangle, \quad (5)$$

其中, $j_2 \oplus j_1$ 表示 j_2+j_1 模2. 以前两个量子比特为控制位, 第三个量子比特为目标位, 3量子比特Toffoli门可

表示为^[60]

$$U_{\text{Tof}} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}. \quad (6)$$

Toffoli门可将3量子比特态 $|j_1 j_2 j_3\rangle$ 转化为相应状态 $(j_1, j_2, j_3=0, 1)$:

$$U_{\text{Tof}}|j_1 j_2 j_3\rangle = |j_1, j_2, j_1 j_2 \oplus j_3\rangle. \quad (7)$$

粒子 A, B, C 分别处于任意单量子比特态:

$$\begin{aligned} |A\rangle &= \alpha_0|0\rangle + \alpha_1|1\rangle, \\ |B\rangle &= \beta_0|0\rangle + \beta_1|1\rangle, \\ |C\rangle &= \gamma_0|0\rangle + \gamma_1|1\rangle. \end{aligned} \quad (8)$$

系数 $\alpha_0, \alpha_1, \beta_0, \beta_1, \gamma_0, \gamma_1$ 满足归一化关系:
 $|\alpha_0|^2 + |\alpha_1|^2 = 1, |\beta_0|^2 + |\beta_1|^2 = 1, |\gamma_0|^2 + |\gamma_1|^2 = 1$. 通讯方 Alice 拥有粒子 A , Bob 拥有粒子 B , Charlie 拥有粒子 C . 通讯方希望执行以粒子 A, B 为控制位, 粒子 C 为目标位的非定域 Toffoli 门.

联合退相位噪声下的非定域 Toffoli 门原理如图 1 所示, Charlie 与通信方 Alice, Bob 共享量子纠缠信道. Alice, Bob 对手中任意量子比特和纠缠粒子执行控制非门操作, 并对手中粒子执行正交投影测量. Charlie 依据正交投影测量结果, 对手中粒子执行相应局域幺正操作, 即可完成非定域 Toffoli 门操作.

假设 Charlie 制备 3 光子纠缠态为部分纠缠态:

$$\begin{aligned} |\psi\rangle_{a_1 a_2 c_1} &= \delta_0|010\rangle + \delta_1|101\rangle, \\ |\psi\rangle_{b_1 b_2 c_2} &= \delta_0|010\rangle + \delta_1|101\rangle, \end{aligned} \quad (9)$$

其中, $|\delta_0|^2 + |\delta_1|^2 = 1, |\delta_0| < |\delta_1|$. 纠缠光子 a_1, a_2 (b_1, b_2) 通过同样的联合退相位噪声信道后, 三光子纠缠态 $|\psi\rangle_{a_1 a_2 c_1}$ ($|\psi\rangle_{b_1 b_2 c_2}$) 形式保持不变. 为实现非定域量子门, 通讯方共享 2 个 3 光子纠缠信道. 即 Charlie 先制备 2 个 3 光子纠缠态 $|\psi\rangle_{a_1 a_2 c_1}, |\psi\rangle_{b_1 b_2 c_2}$, 再分别将纠缠光子 a_1

图1 联合退相位噪声下基于部分纠缠态的非定域 Toffoli 门原理图. 双线表示发送方与接收方之间的经典通讯 $r_1, s_1, r_2, s_2, s'_1, s'_2$ 分别表示粒子 $a_1, a_2, b_1, b_2, c_1, c_2$ 的单粒子测量结果
Figure 1 The schematic principle for implementation of nonlocal Toffoli gate via partially entangled state against collective-dephasing noise. The double lines represent classical information communication from the senders to the receiver and $r_1, s_1, r_2, s_2, s'_1, s'_2$ represent the single-particle measurement results of particles a_1, a_2, b_1, b_2, c_1 , and c_2 .

和 a_2 发送给 Alice, b_1 和 b_2 发送给 Bob.

安全建立量子纠缠信道后, 通信方 Alice, Bob 分别对手中粒子 A, a_1, B, b_1 执行以粒子 A, B 为控制位, a_1, b_1 为目标位的控制非门操作, 并对手中粒子执行单粒子测量. Charlie 依据 Alice, Bob 测量结果执行相应局域幺正操作, 即可完成非定域量子门操作. 由粒子 $A, B, C, a_1, a_2, b_1, b_2$ 所组成复合系统状态可表示为

$$\begin{aligned} |\Psi\rangle &= \sum_{\substack{j_1, j_2, j_3, \\ j_4, j_5=0}}^1 \alpha_{j_1} \beta_{j_2} \gamma_{j_3} \delta_{j_4} \delta_{j_5} |j_1\rangle_A |j_2\rangle_B |j_3\rangle_C \\ &\otimes |j_4, j_4 \oplus 1, j_4\rangle_{a_1 a_2 c_1} |j_5, j_5 \oplus 1, j_5\rangle_{b_1 b_2 c_2}. \end{aligned} \quad (10)$$

Alice, Bob 分别对手中粒子 A, a_1, B, b_1 执行以粒子 A, B 为控制位, a_1, b_1 为目标位的控制非门操作后, 复合系统状态由 $|\Psi\rangle$ 转化为 $|\Psi_1\rangle$:

$$\begin{aligned} |\Psi_1\rangle &= \sum_{\substack{j_1, j_2, j_3, \\ j_4, j_5=0}}^1 \alpha_{j_1} \beta_{j_2} \gamma_{j_3} \delta_{j_4} \delta_{j_5} |j_1\rangle_A |j_2\rangle_B |j_3\rangle_C \\ &\otimes |j_1 \oplus j_4, j_4 \oplus 1, j_4\rangle_{a_1 a_2 c_1} \\ &\otimes |j_2 \oplus j_5, j_5 \oplus 1, j_5\rangle_{b_1 b_2 c_2}. \end{aligned} \quad (11)$$

控制非门操作后, Alice, Bob对粒子 a_1, b_1 执行 $\mathbf{Z}$ 基测量. 剩余粒子 $A, B, C, a_2, b_2, c_1, c_2$ 状态塌缩为 $|\varphi\rangle$, 当粒子 a_1, b_1 的测量结果分别为 r_1, r_2 时($r_1, r_2=0, 1$):

$$|\varphi\rangle = \sum_{j_1, j_2, j_3=0}^1 \alpha_{j_1} \beta_{j_2} \gamma_{j_3} \delta_{r_1 \oplus j_1} \delta_{r_2 \oplus j_2} |j_1\rangle_A |j_2\rangle_B |j_3\rangle_C \\ \otimes |r_1 \oplus j_1 \oplus 1, r_1 \oplus j_1\rangle_{a_2 c_1} \\ \otimes |r_2 \oplus j_2 \oplus 1, r_2 \oplus j_2\rangle_{b_2 c_2}. \quad (12)$$

为实现非定域量子门操作, Alice, Bob对手中纠缠粒子 a_2, b_2 执行 $\mathbf{X}$ 基测量. 由粒子 A, B, C, c_1, c_2 组成复合系统状态塌缩为量子态 $|\varphi_1\rangle$, 当粒子 a_2, b_2 测量结果分别为 s_1, s_2 时($s_1, s_2=0, 1$):

$$|\varphi_1\rangle = \sum_{j_1, j_2, j_3=0}^1 (-1)^{s_1(r_1+j_1+1)} (-1)^{s_2(r_2+j_2+1)} \\ \times \alpha_{j_1} \beta_{j_2} \gamma_{j_3} \delta_{r_1 \oplus j_1} \delta_{r_2 \oplus j_2} \\ \times |j_1\rangle_A |j_2\rangle_B |j_3\rangle_C |r_1 \oplus j_1\rangle_{c_1} |r_2 \oplus j_2\rangle_{c_2}. \quad (13)$$

Charlie依据Alice, Bob测量结果 r_1, r_2, s_1, s_2 对手中粒子 c_1, c_2 分别执行相应局域幺正操作 $\mathbf{Z}^{s_1} \mathbf{X}^{r_1}$ 和 $\mathbf{Z}^{s_2} \mathbf{X}^{r_2}$ 可将复合系统转化为状态 $|\varphi_2\rangle$:

$$(\mathbf{Z}^{s_1} \mathbf{X}^{r_1})_{c_1} \otimes (\mathbf{Z}^{s_2} \mathbf{X}^{r_2})_{c_2} |\varphi_1\rangle = |\varphi_2\rangle, \quad (14)$$

其中,

$$|\varphi_2\rangle = \sum_{j_1, j_2, j_3=0}^1 \alpha_{j_1} \beta_{j_2} \gamma_{j_3} \delta_{r_1 \oplus j_1} \delta_{r_2 \oplus j_2} \\ \times |j_1\rangle_A |j_2\rangle_B |j_3\rangle_C |j_1\rangle_{c_1} |j_2\rangle_{c_2}. \quad (15)$$

为实现非定域量子门, Charlie对她手中粒子 c_1, c_2, C 执行Toffoli门操作. Toffoli操作后复合系统状态演化为 $|\varphi_3\rangle$:

$$|\varphi_3\rangle = \sum_{j_1, j_2, j_3=0}^1 \alpha_{j_1} \beta_{j_2} \gamma_{j_3} \delta_{r_1 \oplus j_1} \delta_{r_2 \oplus j_2} \\ \times |j_1\rangle_A |j_2\rangle_B |j_1 j_2 \oplus j_3\rangle_C |j_1\rangle_{c_1} |j_2\rangle_{c_2}. \quad (16)$$

为消除非最大纠缠信道影响, Charlie先引入一个初态为 $|0\rangle_a$ 的附加量子比特 a , 再对粒子 c_1, c_2, a 执行联合幺正演化并对附加粒子状态进行测量. 与文献[54]类似, 在基 $\{|000\rangle, |010\rangle, |100\rangle, |110\rangle, |001\rangle, |011\rangle, |101\rangle, |111\rangle\}$ 下, 联合幺正演化 $U_{\max}$:

$$U_{\max} = \begin{pmatrix} \frac{\delta_0^2}{\delta_{r_1} \delta_{r_2}} & 0 & 0 & 0 & -\sqrt{1 - \left(\frac{\delta_0^2}{\delta_{r_1} \delta_{r_2}}\right)^2} & 0 & 0 & 0 \\ 0 & \frac{\delta_0^2}{\delta_{r_1} \delta_{r_2 \oplus 1}} & 0 & 0 & 0 & -\sqrt{1 - \left(\frac{\delta_0^2}{\delta_{r_1} \delta_{r_2 \oplus 1}}\right)^2} & 0 & 0 \\ 0 & 0 & \frac{\delta_0^2}{\delta_{r_1 \oplus 1} \delta_{r_2}} & 0 & 0 & 0 & -\sqrt{1 - \left(\frac{\delta_0^2}{\delta_{r_1 \oplus 1} \delta_{r_2}}\right)^2} & 0 \\ 0 & 0 & 0 & \frac{\delta_0^2}{\delta_{r_1 \oplus 1} \delta_{r_2 \oplus 1}} & 0 & 0 & 0 & -\sqrt{1 - \left(\frac{\delta_0^2}{\delta_{r_1 \oplus 1} \delta_{r_2 \oplus 1}}\right)^2} \\ \sqrt{1 - \left(\frac{\delta_0^2}{\delta_{r_1} \delta_{r_2}}\right)^2} & 0 & 0 & 0 & \frac{\delta_0^2}{\delta_{r_1} \delta_{r_2}} & 0 & 0 & 0 \\ 0 & \sqrt{1 - \left(\frac{\delta_0^2}{\delta_{r_1} \delta_{r_2 \oplus 1}}\right)^2} & 0 & 0 & 0 & \frac{\delta_0^2}{\delta_{r_1} \delta_{r_2 \oplus 1}} & 0 & 0 \\ 0 & 0 & \sqrt{1 - \left(\frac{\delta_0^2}{\delta_{r_1 \oplus 1} \delta_{r_2}}\right)^2} & 0 & 0 & 0 & \frac{\delta_0^2}{\delta_{r_1 \oplus 1} \delta_{r_2}} & 0 \\ 0 & 0 & 0 & \sqrt{1 - \left(\frac{\delta_0^2}{\delta_{r_1 \oplus 1} \delta_{r_2 \oplus 1}}\right)^2} & 0 & 0 & 0 & \frac{\delta_0^2}{\delta_{r_1 \oplus 1} \delta_{r_2 \oplus 1}} \end{pmatrix}. \quad (17)$$

可将粒子 A, B, C, c_1, c_2, a 组成的复合系统转化为 $|\Phi\rangle$:

$$|\Phi\rangle = U_{\max} |\varphi_3\rangle_{ABCc_1c_2} |0\rangle_a, \quad (18)$$

其中,

$$|\Phi\rangle = \sum_{j_1, j_2, j_3=0}^1 \alpha_{j_1} \beta_{j_2} \gamma_{j_3} \delta_{r_1 \oplus j_1} \delta_{r_2 \oplus j_2} |j_1\rangle_A \otimes |j_2\rangle_B |j_1 j_2 \oplus j_3\rangle_C |j_1\rangle_{c_1} |j_2\rangle_{c_2} \times \left(\frac{\delta_0^2}{\delta_{r_1 \oplus j_1} \delta_{r_2 \oplus j_2}} |0\rangle + \sqrt{1 - \left(\frac{\delta_0^2}{\delta_{r_1 \oplus j_1} \delta_{r_2 \oplus j_2}} \right)^2} |1\rangle \right). \quad (19)$$

Charlie对附加粒子执行Z基测量, 若测量结果为0则量子态演化成功, 否则量子态演化失败. 成功概率为 $P = 4|\delta_0|^4$. 如果测量结果为0, 粒子A, B, C, c_1, c_2 转化为 $|\phi\rangle$:

$$|\phi\rangle = \sum_{j_1, j_2, j_3=0}^1 \alpha_{j_1} \beta_{j_2} \gamma_{j_3} |j_1\rangle_A |j_2\rangle_B |j_1 j_2 \oplus j_3\rangle_C \otimes |j_1\rangle_{c_1} |j_2\rangle_{c_2}. \quad (20)$$

为完成非定域Toffoli门, Charlie对手中纠缠粒子 c_1, c_2 执行X基测量. 如果测量结果为 s'_1, s'_2 , 则剩余粒子A, B, C状态为

$$|\phi_1\rangle = \sum_{j_1, j_2, j_3=0}^1 (-1)^{j_1 s'_1} (-1)^{j_2 s'_2} \alpha_{j_1} \beta_{j_2} \gamma_{j_3} \times |j_1\rangle_A |j_2\rangle_B |j_1 j_2 \oplus j_3\rangle_C. \quad (21)$$

Alice, Bob分别对手中粒子A, B执行局域么正操作 $Z^{s'_1}$ 和 $Z^{s'_2}$ 即可将粒子A, B, C状态转化为目标态:

$$(Z^{s'_1})_A (Z^{s'_2})_B |\phi_1\rangle = (Z^{s'_1})_A (Z^{s'_2})_B \times \sum_{j_1, j_2, j_3=0}^1 (-1)^{j_1 s'_1} (-1)^{j_2 s'_2} \alpha_{j_1} \beta_{j_2} \gamma_{j_3} \times |j_1\rangle_A |j_2\rangle_B |j_1 j_2 \oplus j_3\rangle_C = \sum_{j_1, j_2, j_3=0}^1 \alpha_{j_1} \beta_{j_2} \gamma_{j_3} |j_1\rangle_A |j_2\rangle_B |j_1 j_2 \oplus j_3\rangle_C. \quad (22)$$

3 联合转动噪声下的非定域Toffoli门概率远程实现

信道联合转动噪声可表示为^[55,56]

$$U_c |0\rangle = \cos\theta |0\rangle + \sin\theta |1\rangle, \\ U_c |1\rangle = -\sin\theta |0\rangle + \cos\theta |1\rangle, \quad (23)$$

其中, θ 为信道联合转动噪声参数.

与联合退相位信道下的非定域量子门远程实现类似, 通信方Alice, Bob, Charlie分别拥有粒子A, B, C. 粒子A, B, C处于任意单量子比特态: $|A\rangle = \alpha_0|0\rangle + \alpha_1|1\rangle$, $|B\rangle = \beta_0|0\rangle + \beta_1|1\rangle$, $|C\rangle = \gamma_0|0\rangle + \gamma_1|1\rangle$. 其中 $|\alpha_0|^2 + |\alpha_1|^2 = 1$, $|\beta_0|^2 + |\beta_1|^2 = 1$, $|\gamma_0|^2 + |\gamma_1|^2 = 1$. Alice, Bob, Charlie希望实现以粒子A, B为控制位, 粒子C为目标位的非定域Toffoli门:

$$U_{\text{Tof}} |A\rangle |B\rangle |C\rangle = U_{\text{Tof}} \sum_{j_1, j_2, j_3=0}^1 \alpha_{j_1} \beta_{j_2} \gamma_{j_3} |j_1\rangle |j_2\rangle |j_3\rangle = \sum_{j_1, j_2, j_3=0}^1 \alpha_{j_1} \beta_{j_2} \gamma_{j_3} |j_1\rangle |j_2\rangle |j_1 j_2 \oplus j_3\rangle. \quad (24)$$

联合转动噪声下非定域Toffoli门原理如图2所示, Alice, Bob和Charlie先共享多粒子纠缠态, Alice和Bob对手中任意量子比特态粒子和纠缠粒子执行控制非门操作, 并对纠缠粒子执行单粒子测量. Charlie依据测量结果对手中粒子执行相应局域么正操作和联合么正演化, 即可实现非定域量子门操作.

假设Charlie制备纠缠态为3粒子部分纠缠态:

$$|\psi\rangle_{a_1 a_2 c_1} = \delta_0 |\varphi_{00}\rangle |0\rangle + \delta_1 |\varphi_{11}\rangle |1\rangle, \\ |\psi\rangle_{b_1 b_2 c_2} = \delta_0 |\varphi_{00}\rangle |0\rangle + \delta_1 |\varphi_{11}\rangle |1\rangle, \quad (25)$$

其中, $|\delta_0|^2 + |\delta_1|^2 = 1$, $|\delta_0| < |\delta_1|$.

$$|\varphi_{00}\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle), \\ |\varphi_{11}\rangle = \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle). \quad (26)$$

光子 a_1, a_2 (b_1, b_2)通过相同联合转动信道, 纠缠态 $|\psi\rangle_{a_1 a_2 c_1}, |\psi\rangle_{b_1 b_2 c_2}$ 形式保持不变. Charlie将粒子 a_1, a_2 发送给Alice, 粒子 b_1, b_2 发送给Bob, 保留粒子 c_1, c_2 . 粒子A, B, C, $a_1, a_2, c_1, b_1, b_2, c_2$ 组成复合系统状态可表示为

$$|\Psi\rangle = |A\rangle \otimes |B\rangle \otimes |C\rangle \otimes |\psi\rangle_{a_1 a_2 c_1} \otimes |\psi\rangle_{b_1 b_2 c_2} = \sum_{\substack{j_1, j_2, j_3, j_4 \\ j_5, j_6, j_7=0}}^1 (-1)^{j_4 j_5} (-1)^{j_6 j_7} \alpha_{j_1} \beta_{j_2} \gamma_{j_3} \delta_{j_4} \delta_{j_6} |j_1\rangle |j_2\rangle \otimes |j_3\rangle |j_5\rangle |j_5 \oplus j_4\rangle |j_4\rangle |j_7\rangle |j_7 \oplus j_6\rangle |j_6\rangle. \quad (27)$$

Alice和Bob分别以粒子 A, B 为控制位, 粒子 a_1, b_1 为目标位执行控制非门操作. 复合系统状态由 $|\psi\rangle$ 转化为

$$|\psi\rangle = \sum_{\substack{j_1, j_2, j_3, j_4 \\ j_5, j_6, j_7=0}} (-1)^{j_4 j_5} (-1)^{j_6 j_7} \alpha_{j_1} \beta_{j_2} \gamma_{j_3} \delta_{j_4} \delta_{j_6} |j_1\rangle |j_2\rangle |j_3\rangle \\ \otimes |j_5 \oplus j_1\rangle |j_6 \oplus j_4\rangle |j_7 \oplus j_2\rangle |j_7 \oplus j_6\rangle |j_6\rangle. \quad (28)$$

Alice, Bob对粒子 a_1, a_2, b_1, b_2 执行 $\mathbf{Z}$ 基测量, 并公布测量结果, Charlie依据Alice, Bob测量结果对手中纠缠粒子 c_1, c_2 执行相应局域幺正操作. 假设粒子 a_1, a_2, b_1, b_2 测量结果分别为 $s_{11}, s_{12}, s_{21}, s_{22}$ ($s_{11}, s_{12}, s_{21}, s_{22}=0, 1$), 则剩余粒子 A, B, C, c_1, c_2 状态塌缩为

$$|\xi\rangle = \sum_{j_1, j_2, j_3=0}^1 (-1)^{j_1 \oplus s_{11} \oplus s_{12} (j_1 \oplus s_{11})} \\ \times (-1)^{j_2 \oplus s_{21} \oplus s_{22} (j_2 \oplus s_{21})} \\ \otimes \alpha_{j_1} \beta_{j_2} \gamma_{j_3} \delta_{j_1 \oplus s_{11} \oplus s_{12}} \delta_{j_2 \oplus s_{21} \oplus s_{22}} |j_1\rangle |j_2\rangle |j_3\rangle \\ \otimes |j_1 \oplus s_{11} \oplus s_{12}\rangle |j_2 \oplus s_{21} \oplus s_{22}\rangle. \quad (29)$$

Charlie依据测量结果 $s_{11}, s_{12}, s_{21}, s_{22}$ 对粒子 c_1, c_2 选择相应的局域幺正操作 $\mathbf{U}_{s_{11}, s_{12}}$, $\mathbf{U}_{s_{21}, s_{22}}$ 可将复合系统状态转化为 $|\xi_1\rangle$:

$$|\xi_1\rangle = (\mathbf{U}_{s_{11}, s_{12}})_{c_1} (\mathbf{U}_{s_{21}, s_{22}})_{c_2} |\xi\rangle, \quad (30)$$

其中,

$$|\xi_1\rangle = \sum_{j_1, j_2, j_3=0}^1 \alpha_{j_1} \beta_{j_2} \gamma_{j_3} \delta_{j_1 \oplus s_{11} \oplus s_{12}} \delta_{j_2 \oplus s_{21} \oplus s_{22}} \\ \times |j_1\rangle |j_2\rangle |j_3\rangle |j_1\rangle |j_2\rangle, \\ \mathbf{U}_{s_{11}, s_{21}} = \sum_{l_1=0}^1 (-1)^{(l_1 \oplus s_{11} \oplus s_{21}) (l_1 \oplus s_{11})} |l_1\rangle \langle l_1 \oplus s_{11} \oplus s_{21}|, \quad (31)$$

式中, $s_1, s_2=0, 1$.

为实现非定域量子门, Charlie先对粒子 c_1, c_2, C 执行Toffoli门, 再对粒子 c_1, c_2 执行 $\mathbf{X}$ 基测量. Toffoli门将粒子 A, B, C, c_1, c_2 状态转化为

$$|\xi_2\rangle = U_{\text{Toff}} |\xi_1\rangle \\ = \sum_{j_1, j_2, j_3=0}^1 \alpha_{j_1} \beta_{j_2} \gamma_{j_3} \delta_{j_1 \oplus s_{11} \oplus s_{12}} \delta_{j_2 \oplus s_{21} \oplus s_{22}} \\ \times |j_1\rangle |j_2\rangle |j_1 j_2 \oplus j_3\rangle |j_1\rangle |j_2\rangle. \quad (32)$$

为消除非最大纠缠信道对非定域量子门影响, Charlie先引入一个初态为 $|0\rangle_a$ 的附加量子比特 a , 再对粒子 c_1, c_2, a 执行联合幺正演化并对附加粒子状态进行测量. 与文献[57]类似, 在基 $\{|000\rangle, |010\rangle, |100\rangle, |110\rangle, |001\rangle, |011\rangle, |101\rangle, |111\rangle\}$ 下, 联合幺正演化 $\mathbf{U}'_{\max}$ 可表示为

$$\mathbf{U}'_{\max} = \begin{bmatrix} \Gamma_{00} & 0 & 0 & 0 & -\sqrt{1-(\Gamma_{00})^2} & 0 & 0 & 0 \\ 0 & \Gamma_{01} & 0 & 0 & 0 & -\sqrt{1-(\Gamma_{01})^2} & 0 & 0 \\ 0 & 0 & \Gamma_{10} & 0 & 0 & 0 & -\sqrt{1-(\Gamma_{10})^2} & 0 \\ 0 & 0 & 0 & \Gamma_{11} & 0 & 0 & 0 & -\sqrt{1-(\Gamma_{11})^2} \\ \sqrt{1-(\Gamma_{00})^2} & 0 & 0 & 0 & \Gamma_{00} & 0 & 0 & 0 \\ 0 & \sqrt{1-(\Gamma_{01})^2} & 0 & 0 & 0 & \Gamma_{01} & 0 & 0 \\ 0 & 0 & \sqrt{1-(\Gamma_{10})^2} & 0 & 0 & 0 & \Gamma_{10} & 0 \\ 0 & 0 & 0 & \sqrt{1-(\Gamma_{11})^2} & 0 & 0 & 0 & \Gamma_{11} \end{bmatrix}, \quad (33)$$

其中, $\Gamma_{j_1 j_2} = \frac{\delta_0^2}{\delta_{j_1 \oplus s_{11} \oplus s_{12}} \delta_{j_2 \oplus s_{21} \oplus s_{22}}}$, ($j_1, j_2 = 0, 1$).

联合幺正演化可将粒子 A, B, C, c_1, c_2, a 所组成复合系统状态转化为相应量子态:

图2 联合转动噪声下基于部分纠缠态的非定域Toffoli门原理图. $s_{11}, s_{12}, s_{21}, s_{22}, s'_1, s'_2$ 分别表示粒子 $a_1, a_2, b_1, b_2, c_1, c_2$ 的单粒子测量结果

Figure 2 The schematic principle for implementation of nonlocal Toffoli gate via partially entangled state against collective-rotating noise. $s_{11}, s_{12}, s_{21}, s_{22}, s'_1, s'_2$ represent the single-particle measurement results of particles a_1, a_2, b_1, b_2, c_1 , and c_2 .

$$|\Phi\rangle = U'_{\max} |\xi_2\rangle_{ABCc_1c_2} |0\rangle_a, \quad (34)$$

式中,

$$\begin{aligned} |\Phi\rangle = & \sum_{j_1, j_2, j_3=0}^1 \alpha_{j_1} \beta_{j_2} \gamma_{j_3} \delta_{j_1 \oplus s_{11} \oplus s_{12} \oplus s_{21} \oplus s_{22}} \\ & \times |j_1\rangle_A |j_2\rangle_B |j_1 j_2 \oplus j_3\rangle_C |j_1\rangle_{j_2} \\ & \otimes \left[\frac{\delta_0^2}{\delta_{j_1 \oplus s_{11} \oplus s_{12} \oplus s_{21} \oplus s_{22}}} |0\rangle \right. \\ & \left. + \sqrt{1 - \left(\frac{\delta_0^2}{\delta_{j_1 \oplus s_{11} \oplus s_{12} \oplus s_{21} \oplus s_{22}}} \right)^2} |1\rangle \right]_a. \end{aligned} \quad (35)$$

么正演化后, Charlie对附加粒子执行 Z 基测量, 若测量结果为0, 则演化成功, 否则演化失败. 成功率为 $P = 4|\delta_0|^4$. 若附加粒子测量结果为0, 则粒子 A, B, C, c_1, c_2 转化为

$$\begin{aligned} |\phi\rangle = & \sum_{j_1, j_2, j_3=0}^1 \alpha_{j_1} \beta_{j_2} \gamma_{j_3} |j_1\rangle_A |j_2\rangle_B |j_1 j_2 \oplus j_3\rangle_C \\ & \otimes |j_1\rangle_{c_1} |j_2\rangle_{c_2}. \end{aligned} \quad (36)$$

与联合退相位信道下的非定域量子门类似, Charlie对粒子 c_1, c_2 执行 X 基测量, Alice, Bob依据测量结果执行相应局域么正操作, 即可实现3粒子非定域Toffoli

门. 假设 X 基测量结果为 s'_1, s'_2 , 则粒子 A, B, C 状态为

$$\begin{aligned} |\phi_1\rangle = & \sum_{j_1, j_2, j_3=0}^1 (-1)^{j_1 s'_1} (-1)^{j_2 s'_2} \alpha_{j_1} \beta_{j_2} \gamma_{j_3} \\ & \times |j_1\rangle_A |j_2\rangle_B |j_1 j_2 \oplus j_3\rangle_C. \end{aligned} \quad (37)$$

局域么正操作 $Z^{s'_1}$ 和 $Z^{s'_2}$ 可将粒子 A, B, C 状态转化为目标态:

$$\begin{aligned} & (Z^{s'_1})_A (Z^{s'_2})_B |\phi_1\rangle \\ = & (Z^{s'_1})_A (Z^{s'_2})_B \otimes \sum_{j_1, j_2, j_3=0}^1 (-1)^{j_1 s'_1} (-1)^{j_2 s'_2} \alpha_{j_1} \beta_{j_2} \gamma_{j_3} \\ & \times |j_1\rangle_A |j_2\rangle_B |j_1 j_2 \oplus j_3\rangle_C \\ = & \sum_{j_1, j_2, j_3=0}^1 \alpha_{j_1} \beta_{j_2} \gamma_{j_3} |j_1\rangle_A |j_2\rangle_B |j_1 j_2 \oplus j_3\rangle_C. \end{aligned} \quad (38)$$

4 结论

在量子信息实际应用中, 一方面非定域Toffoli门远程实现成功率与大尺度分布式量子计算效率密切相关; 另一方面纠缠态是量子信息处理的珍贵资源, 在非定域Toffoli门远程实现中应尽量提高量子纠缠态使用率. 在我们提出的联合噪声下基于部分纠缠信道的非定域Toffoli门远程实现方案中, 非定域Toffoli门远程实现成功率等于信道纠缠度, 因而方案是最优的. 在基于部分纠缠态的非定域Toffoli门远程实现方案中, 通讯方也可选择对手中部分纠缠粒子 $|\psi\rangle_{a_1 a_2 c_1} = \delta_0 |010\rangle + \delta_1 |101\rangle$ 执行相应半正定算子测量. 这种方法在提高非定域Toffoli门远程实现成功率的同时也需要消耗更多的量子纠缠资源^[61].

当系数 $\delta_0 = \delta_1 = \frac{1}{\sqrt{2}}$, 量子纠缠信道由2个3粒子最大纠缠态组成. 与文献[62]类似, 所有通信方合作可实现确定的非定域Toffoli门操作. 演化矩阵 $U'_{\max}$ 为单位矩阵 $I_{8 \times 8}$, 即Charlie不需要对手中粒子 c_1, c_2 和附加粒子 a 执行联合么正演化, 只需与Alice, Bob合作对粒子 c_1, c_2 执行相应单粒子操作, 对粒子 c_1, c_2, C 执行局域Toffoli门操作, 就可以完成非定域量子门操作.

与联合退相位噪声下的非定域Toffoli门远程实现类似, 通讯方可用4量子比特退相关无关子空间同时克制信道联合退相位噪声以及信道联合转动噪声影响.

与文献[63,64]类似, Charlie制备纠缠态为5光子部分纠缠态:

$$\begin{aligned} |\psi\rangle_{a_1 a_2 a_3 a_4 c_1} &= \delta_0 |\eta_0\rangle|0\rangle + \delta_1 |\eta_1\rangle|0\rangle, \\ |\psi\rangle_{b_1 b_2 b_3 b_4 c_2} &= \delta_0 |\eta_0\rangle|0\rangle + \delta_1 |\eta_1\rangle|0\rangle, \end{aligned} \quad (39)$$

其中,

$$\begin{aligned} |\eta_0\rangle &= \frac{1}{2}(|0101\rangle - |0110\rangle - |1001\rangle + |1010\rangle), \\ |\eta_1\rangle &= \frac{1}{2\sqrt{3}}(2|0011\rangle - |0101\rangle - |0110\rangle - |1001\rangle \\ &\quad - |1010\rangle + 2|0110\rangle). \end{aligned} \quad (40)$$

纠缠光子 a_1, a_2, a_3, a_4 (b_1, b_2, b_3, b_4)通过相同联合噪声信道, 纠缠态 $|\psi\rangle$ 保持形式不变. 与联合转动噪声信道下的非定域Toffoli门远程实现类似, Alice(Bob)分

别对粒子 A, a_1 (B, b_1)执行控制非门操作, 并对纠缠粒子执行相应单粒子测量. Charlie依据Alice(Bob)测量结果选择执行相应局域么正操作和单粒子测量, 即可实现非定域Toffoli门操作.

本文以部分纠缠态为纠缠信道, 提出了2个联合噪声信道下的非定域Toffoli门远程实现方案, 一个方案用于联合退相位噪声下基于部分纠缠信道的非定域Toffoli门远程实现, 另一个方案用于联合转动噪声下基于部分纠缠信道的非定域Toffoli门远程实现. 通讯方先用退相干无关子空间克制信道联合噪声影响, 再通过引入附加粒子和执行联合么正演化消除部分纠缠信道对非定域Toffoli门远程实现影响. 由于方案不需要使用最大纠缠态来实现非定域Toffoli门, 与其他方案相比, 具有可行性强的优点.

参考文献

- Nielsen M A, Chuang I L. Quantum Computation and Quantum Information. 2nd ed. London: Cambridge University Press, 2010. 26–28
- Bennett C H, Brassard G, Crepeau C, et al. Teleporting an unknown quantum state via dual classical and Einstein-Podolsky-Rosen channels. *Phys Rev Lett*, 1993, 70: 1895–1899
- Long G L, Liu X S. Theoretically efficient high-capacity quantum-key-distribution scheme. *Phys Rev A*, 2002, 65: 032302
- Zhang W, Ding D S, Sheng Y B, et al. Quantum secure direct communication with quantum memory. *Phys Rev Lett*, 2017, 118: 220501, arXiv: 1609.09184
- Zhu F, Zhang W, Sheng Y, et al. Experimental long-distance quantum secure direct communication. *Sci Bull*, 2017, 62: 1519–1524
- Deng F G, Long G L, Liu X S. Two-step quantum direct communication protocol using the Einstein-Podolsky-Rosen pair block. *Phys Rev A*, 2003, 68: 042317
- Deng F G, Long G L. Secure direct communication with a quantum one-time pad. *Phys Rev A*, 2004, 69: 052319
- Zheng C, Long G F. Quantum secure direct dialogue using Einstein-Podolsky-Rosen pairs. *Sci China-Phys Mech Astron*, 2014, 57: 1238–1243
- Hu J Y, Yu B, Jing M Y, et al. Experimental quantum secure direct communication with single photons. *Light Sci Appl*, 2016, 5: e16144, arXiv: 1503.00451
- Karlsson A, Bourennane M. Quantum teleportation using three-particle entanglement. *Phys Rev A*, 1998, 58: 4394–4400
- Bouwmeester D, Pan J W, Mattle K, et al. Experimental quantum teleportation. *Nature*, 1997, 390: 575–579, arXiv: 1901.11004
- Xiao X, Yao Y, Zhong W J, et al. Enhancing teleportation of quantum Fisher information by partial measurements. *Phys Rev A*, 2016, 93: 012307, arXiv: 1510.07359
- Zha X W, Zhang C M. Teleportation of on N -particle GHZ state via one three-particle W state (in Chinese). *Acta Phys Sin*, 2008, 57: 1339–1342 [查新未, 张淳民. 利用一个三粒子W态隐形传送 N 粒子GHZ态. *物理学报*, 2008, 57: 1339–1342]
- Yan L, Ma H Y, Zheng C, et al. Quantum communication scheme based on quantum teleportation (in Chinese). *Acta Phys Sin*, 2017, 66: 230303 [杨璐, 马鸿洋, 郑超, 等. 基于量子隐形传态的量子保密通信方案. *物理学报*, 2017, 66: 230303]
- Deng F G, Li C Y, Li Y S, et al. Symmetric multiparty-controlled teleportation of an arbitrary two-particle entanglement. *Phys Rev A*, 2005, 72: 022338
- Pati A K. Minimum classical bit for remote preparation and measurement of a qubit. *Phys Rev A*, 2000, 63: 014302
- Lo H K. Classical-communication cost in distributed quantum-information processing: A generalization of quantum-communication complexity. *Phys Rev A*, 2000, 62: 012313

- 18 Bennett C H, DiVincenzo D P, Shor P W, et al. Remote state preparation. *Phys Rev Lett*, 2001, 87: 077902
- 19 Xia Y, Song J, Song H S. Multiparty remote state preparation. *J Phys B-At Mol Opt Phys*, 2007, 40: 3719–3724
- 20 Yan A. Bidirectional controlled teleportation via six-qubit cluster state. *Int J Theor Phys*, 2013, 52: 3870–3873
- 21 Li Y, Nie L, Li X, et al. Asymmetric bidirectional controlled teleportation by using six-qubit cluster state. *Int J Theor Phys*, 2016, 55: 3008–3016
- 22 Yang Y Q, Zha X W, Yu Y. Asymmetric bidirectional controlled teleportation via seven-qubit cluster state. *Int J Theor Phys*, 2016, 55: 4197–4204
- 23 Liang H Q, Liu J M, Feng S S, et al. Effects of noises on joint remote state preparation via a GHZ-class channel. *Quantum Inf Process*, 2015, 14: 3857–3877
- 24 Zhou P, Jiao X F, Lv S X. Parallel remote state preparation of arbitrary single-qubit states via linear-optical elements by using hyperentangled Bell states as the quantum channel. *Quantum Inf Process*, 2018, 17: 298
- 25 Huelga S F, Vaccaro J A, Chefles A, et al. Quantum remote control: Teleportation of unitary operations. *Phys Rev A*, 2001, 63: 042303
- 26 He Y H, Lu Q C, Liao Y M, et al. Bidirectional controlled remote implementation of an arbitrary single qubit unitary operation with EPR and cluster states. *Int J Theor Phys*, 2015, 54: 1726–1736
- 27 Fan Q B, Liu D D. Controlled remote implementation of partially unknown quantum operation. *Sci China Ser G-Phys Mech Astron*, 2008, 51: 1661–1667
- 28 Lin J Y, He J G, Gao Y C, et al. Controlled remote implementation of an arbitrary single-qubit operation with partially entangled quantum channel. *Int J Theor Phys*, 2017, 56: 1085–1095
- 29 Wang S, Liu Y, Chen J, et al. Deterministic single-qubit operation sharing with five-qubit cluster state. *Quantum Inf Process*, 2013, 12: 2497–2507
- 30 Peng J. Tripartite operation sharing with five-qubit Brown state. *Quantum Inf Process*, 2016, 15: 2465–2473
- 31 Tian D P, Tao Y J, Qin M. Teleportation of an arbitrary two-qudit state based on the non-maximally four-qudit cluster state. *Sci China Ser G-Phys Mech Astron*, 2008, 51: 1523–1528
- 32 Lv S X, Zhao Z W, Zhou P. Joint remote control of an arbitrary single-qubit state by using a multiparticle entangled state as the quantum channel. *Quantum Inf Process*, 2018, 17: 8
- 33 Barenco A, Bennett C H, Cleve R, et al. Elementary gates for quantum computation. *Phys Rev A*, 1995, 52: 3457–3467
- 34 Ye M Y, Zhang Y S, Guo G C. Quantum entanglement and quantum operation (in Chinese). *Sci Sin-Phys Mech Astron*, 2007, 37: 716–722 [叶明勇, 张永生, 郭光灿. 量子纠缠和量子操作. 中国科学: 物理学 力学 天文学, 2007, 37: 716–722]
- 35 Chen L B, Lu H. Conclusive qudit states transformation implemented by remote parties (in Chinese). *Sci Sin-Phys Mech Astron*, 2015, 45: 120301 [陈立冰, 路洪. 远程协作实现高维量子态间的精确转换. 中国科学: 物理学 力学 天文学, 2015, 45: 120301]
- 36 Chen L B, Lu H. Deterministic and controlled many-to-one and one-to-many remote quantum rotations via partially entangled quantum channels (in Chinese). *Sci Sin-Phys Mech Astron*, 2014, 44: 1187–1195 [陈立冰, 路洪. 利用部分纠缠的量子信道确定性地实现 $N \rightarrow 1$ 和 $1 \rightarrow N$ 的受控量子远程旋转. 中国科学: 物理学 力学 天文学, 2014, 44: 1187–1195]
- 37 Xiang G Y, Li J, Guo G C. Teleporting a rotation on remote photons. *Phys Rev A*, 2005, 71: 044304
- 38 Huang Y F, Ren X F, Zhang Y S, et al. Experimental teleportation of a quantum controlled-NOT gate. *Phys Rev Lett*, 2004, 93: 240501
- 39 Eisert J, Jacobs K, Papadopoulos P, et al. Optimal local implementation of nonlocal quantum gates. *Phys Rev A*, 2000, 62: 052317
- 40 Huelga S F, Plenio M B, Vaccaro J A. Remote control of restricted sets of operations: Teleportation of angles. *Phys Rev A*, 2002, 65: 042316
- 41 Chen L B, Lu H. Nonlocal multi-target controlled-controlled gate using Greenberger-Horne-Zeilinger channel and qutrit catalysis. *Chin Phys B*, 2015, 24: 070307
- 42 Chen L B, Lu H. Quantum networks for implementing locally and conclusively a nonlocal qudit Toffoli gate: Designing and optimizing (in Chinese). *Sci Sin-Phys Mech Astron*, 2016, 46: 110301 [陈立冰, 路洪. 确定性地实现非局域高维量子Toffoli门的网络设计与优化. 中国科学: 物理学 力学 天文学, 2016, 46: 110301]
- 43 Chen L B, Lu H. Efficient nonlocal m -control and n -target controlled unitary gate using non-symmetric GHZ states. *Int J Theor Phys*, 2018, 57: 706–714
- 44 Chou K S, Blumoff J Z, Wang C S, et al. Deterministic teleportation of a quantum gate between two logical qubits. *Nature*, 2018, 561: 368–373
- 45 Bennett C H, Brassard G, Popescu S, et al. Purification of noisy entanglement and faithful teleportation via noisy channels. *Phys Rev Lett*, 1996, 76: 722–725

- 46 Sheng Y B, Deng F G. Deterministic entanglement purification and complete nonlocal Bell-state analysis with hyperentanglement. *Phys Rev A*, 2010, 81: 032307, arXiv: [0912.0079](#)
- 47 Sheng Y B, Deng F G. One-step deterministic polarization-entanglement purification using spatial entanglement. *Phys Rev A*, 2010, 82: 044305, arXiv: [1008.3509](#)
- 48 Li X H, Deng F G, Zhou H Y. Faithful qubit transmission against collective noise without ancillary qubits. *Appl Phys Lett*, 2007, 91: 144101, arXiv: [0708.0068](#)
- 49 Walton Z D, Abouraddy A F, Sergienko A V, et al. Decoherence-free subspaces in quantum key distribution. *Phys Rev Lett*, 2003, 91: 087901
- 50 Deng F G, Li X H, Li T. Quantum error rejection and fault tolerant quantum communication (in Chinese). *Acta Phys Sin*, 2018, 67: 130301 [邓富国, 李熙涵, 李涛. 基于光量子态避错及容错传输的量子通信. *物理学报*, 2018, 67: 130301]
- 51 Yamamoto T, Shimamura J, Özdemir S K, et al. Faithful qubit distribution assisted by one additional qubit against collective noise. *Phys Rev Lett*, 2005, 95: 040503
- 52 Li X H, Deng F G, Zhou H Y. Efficient quantum key distribution over a collective noise channel. *Phys Rev A*, 2008, 78: 022321, arXiv: [0808.0042](#)
- 53 Dong L, Xiu X M, Gao Y J. Three-party controlled quantum teleportation with six-photon entangled states via collective noise channel. *Acta Phys Pol B*, 2010, 41: 2377–2385
- 54 Gu B, Pei S X, Song B, et al. Secure quantum communication under joint noise (in Chinese). *Sci Sin-Phys Mech Astron*, 2010, 40: 177–182 [顾斌, 裴世鑫, 宋标, 等. 联合噪声下确定的安全量子通信. *中国科学: 物理学 力学 天文学*, 2010, 40: 177–182]
- 55 Li Z, Long L R, Zhou P, et al. Probabilistic multiparty-controlled teleportation of an arbitrary m -qubit state with a pure entangled quantum channel against collective noise. *Sci China-Phys Mech Astron*, 2012, 55: 2445–2451
- 56 Zhou P, Li H W, Long L R. Probabilistic multiparty joint remote preparation of an arbitrary m -qubit state with a pure entangled channel against collective noise. *Int J Theor Phys*, 2013, 52: 849–861
- 57 Takeuchi Y, Fujii K, Ikuta R, et al. Blind quantum computation over a collective-noise channel. *Phys Rev A*, 2016, 93: 052307, arXiv: [1505.04248](#)
- 58 Sheng Y B, Zhou L. Blind quantum computation with a noise channel. *Phys Rev A*, 2018, 98: 052343
- 59 Wang H F, Zhu A D, Zhang S, et al. Optically controlled phase gate and teleportation of a controlled-not gate for spin qubits in a quantum-dot-microcavity coupled system. *Phys Rev A*, 2013, 87: 062337, arXiv: [1306.4737](#)
- 60 Hu S, Cui W X, Wang D Y, et al. Teleportation of a Toffoli gate among distant solid-state qubits with quantum dots embedded in optical microcavities. *Sci Rep*, 2015, 5: 11321
- 61 Chen L B, Lu H. Implementing a nonlocal Toffoli gate using partially entangled qubit pairs. *Int J Theor Phys*, 2011, 50: 3442–3450
- 62 Zhou P, Li X H, Deng F G, et al. Multiparty-controlled teleportation of an arbitrary m -qudit state with a pure entangled quantum channel. *J Phys A-Math Theor*, 2007, 40: 13121–13130
- 63 Bourennane M, Eibl M, Gaertner S, et al. Decoherence-free quantum information processing with four-photon entangled states. *Phys Rev Lett*, 2004, 92: 107901
- 64 Tsai C L, Hwang T. Semi-quantum key distribution robust against combined collective noise. *Int J Theor Phys*, 2018, 57: 3410–3418

Probabilistic remote implementation of nonlocal Toffoli gate with pure entangled states against collective noise

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Remote implementation of nonlocal quantum gates is one of the central tasks in quantum communication. Researchers devote much interest in remote implementation of nonlocal quantum gates via quantum entangled channel. Remote implementation protocols of two-qubit controlled-not gate, three-qubit Toffoli gate have been presented. We present two schemes for remote implementation of nonlocal Toffoli gate via partially entangled quantum channel against collective noise. The first is used to remote implementing of nonlocal Toffoli gate via partially entangled channel against collective-dephasing noise, and the second is used to remote implementing of nonlocal Toffoli gate against collective-rotation noise. The schemes are tolerant of collective-dephasing noise and collective-rotating noise. The agents can implement the Toffoli gate nonlocally with partially entangled quantum channel by introducing an auxiliary qubit and performing general evolution. The schemes are more convenient in application since they only requires partially entangled quantum states for nonlocal implementation of quantum operations.

nonlocal Toffoli gate, collective-dephasing noise, collective-rotation noise, partially entangled states

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