基于原子-腔系统的非绝热和乐量子计算

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摘要 量子计算的实际应用依赖于高保真度的量子门, 而获得高保真度量子门所面临的主要挑战之一是系统的控制误差. 几何相具有仅依赖系统演化路径而与演化速度大小等细节无关的特点, 因此基于几何相设计的量子门具有抵抗系统控制误差的抗噪声性. 特别是, 基于非绝热非阿贝尔几何相设计的和乐量子门具有完全的几何性质, 并且不受绝热缓慢演化条件的限制, 受到了人们的广泛关注. 本文提出了利用原子-腔系统实现非绝热和乐量子计算的方案. 以四能级原子的基态作为逻辑量子比特编码空间的基矢, 在激光脉冲的操控下, 通过公共腔模交换虚光子产生双原子基态和辅助状态之间的跃迁, 实现了两比特非绝热和乐受控相位门, 它与通过激光脉冲操控单原子实现的任意单比特非绝热和乐门一起组成了非绝热和乐量子计算的通用量子门.

关键词 量子计算, 几何相, 和乐量子计算, 原子-腔系统

量子计算具有超越经典计算机的运算能力, 能够解决经典计算机不能解决的困难问题, 比如大整数分解问题^[1]和非结构化数据的搜索问题^[2], 但量子计算机的实际应用依赖于高保真度的量子逻辑门, 而实现高保真度量子门所面临的主要挑战之一是系统的控制误差. 由于几何相——包括Berry相^[3]、绝热非阿贝尔几何相^[4]、非绝热几何相^[5]和非绝热非阿贝尔几何相^[6]——具有仅依赖系统的演化路径而与演化速度大小等细节无关的特点, 因此基于几何相设计的量子门具有抵抗系统控制误差的抗噪声性. 关于几何量子计算, 人们最早提出的是基于Berry相的绝热几何量子计算^[7], 随后逐步发展提出了基于绝热非阿贝尔几何相的绝热和乐量子计算^[8,9]、基于非绝热几何相的非绝热几何量子计算^[10,11]和基于非绝热非阿贝尔几何相的非绝热和乐量子计算^[12,13]. 与绝热几何或绝热和乐量子计算相比, 非绝热和乐量子计算不受绝热缓慢演化条件的限制, 可以实现快速演化; 与非绝热几何量子计算相比,

非绝热和乐量子计算的通用量子门具有全几何性质, 即演化算符矩阵整体是几何性的, 而非绝热几何量子门的几何性仅表现在相位参数上.

由于非绝热和乐量子计算具有完全的几何性质, 并且可以快速演化, 因而受到了人们的广泛关注. 自2012年首次提出非绝热和乐量子计算^[12,13]以来, 非绝热和乐量子计算在理论和实验两方面都有了很大的进展. 基于各类不同的量子系统, 人们提出了非绝热和乐量子计算的多种理论方案^[14-42], 并在包括核磁共振系统、金刚石色心系统和超导系统在内的多个实际物理系统中完成了实验演示^[43-51]. 非绝热和乐量子计算的进展还体现在量子门基本设计理论的改进和推广上, 人们先后提出了和乐量子门的一步实现方案^[21,23]、单路径多脉冲方案^[27]和复合门方案^[28], 并把无退相干子空间的非绝热和乐量子计算推广到了无噪声子体系^[17]. 特别是, 一步实现方案简化了单比特门的操作, 使得原方案中需要通过结合两个不对易的 π 旋转

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门而实现任意单比特门的操作简化为通过一步操作即可实现.

非绝热和乐量子计算^[12,13]的实现依赖于满足周期性演化条件和并行移动条件的量子态子空间. 考虑一个由哈密顿量 $H(t)$ 控制的 N 维量子系统, 其演化算符为 $U(t) = T\exp[-i\int_0^t H(t')dt']$. 用 $\{|\phi_1(0)\rangle, |\phi_2(0)\rangle, \dots, |\phi_L(0)\rangle\}$ 来表示 L 维子空间的一组标准正交基, 该组基矢随时间的演化状态可通过演化算符表示为 $|\phi_k(t)\rangle = U(t)|\phi_k(0)\rangle$ ($k = 1, 2, \dots, L$). 如果这个 L 维子空间中的量子态的演化满足以下两个分别被称为周期性演化条件和并行移动条件的表达式^[13]:

$$\sum_{k=1}^L |\phi_k(\tau)\rangle\langle\phi_k(\tau)| = \sum_{k=1}^L |\phi_k(0)\rangle\langle\phi_k(0)|, \quad (1)$$

$$\langle\phi_k(t)|H(t)|\phi_l(t)\rangle = 0, \quad k, l = 1, 2, \dots, L, \quad (2)$$

那么, 当把量子比特编码在子空间 $S^L = \text{Span}\{|\phi_1(0)\rangle, |\phi_2(0)\rangle, \dots, |\phi_L(0)\rangle\}$ 内, 作用在 S^L 上的酉演化算符 $U(\tau)$ 就是一个非绝热和乐门. 因此, 非绝热和乐量子计算的物理实现, 就是发现或构造具有满足上述条件的子空间的物理系统, 在该子空间内设计出一组包括任意单比特门和两比特纠缠门的普适量子门.

本文考虑利用原子-腔系统实现非绝热和乐量子计算. 原子与腔场的耦合是实现光与物质相互作用的重要物理系统^[52,53], 当把两个原子置于同一腔场时, 随着虚光子的交换, 可以使耦合到公共腔上的两个原子(量子比特)之间存在相互作用^[54,55], 从而可实现两比特量子门操作. 在本文之前, 人们已经提出了一个基于原子-腔系统实现非绝热和乐量子计算的方案^[31], 由于该方案所给出的两比特受控门利用了腔模的状态作为辅助状态, 因而对腔模的衰减比较敏感. 在这里, 我们给出基于原子-腔系统实现非绝热和乐量子计算的一个新方案, 这一方案的主要进步体现在两比特受控门的设计上: 利用腔模耦合两个四能级原子, 在激光的操控下, 通过公共腔模交换虚光子实现双原子基态和辅助状态之间的跃迁, 同时把有效的双原子跃迁从公共的腔模中分离出来, 使得两比特受控门对腔模的衰减不敏感. 对于任意单比特门的设计, 利用非共振激光产生的斯塔克能移替代了原始一步方案中的失谐量, 使得仅需改变激光器的拉比频率和相位就可以一步实现任意的单比特非绝热和乐门.

1 单比特非绝热和乐门

非绝热和乐量子计算的普适门由任意单比特门和一个两比特纠缠门组成, 因此在给出本文的主要工作两比特受控门之前, 首先给出任意单比特门的一步实现方案. 考虑一个四能级原子, 它有三个稳定的基态, 记为 $|0\rangle$ 、 $|1\rangle$ 、 $|2\rangle$, 和一个激发态 $|e\rangle$. 量子态 $|e\rangle \leftrightarrow |0\rangle$ 和 $|e\rangle \leftrightarrow |1\rangle$ 之间的跃迁分别由拉比频率为 Ω_{e0} 和 Ω_{e1} 的共振激光驱动, 量子态 $|e\rangle \leftrightarrow |2\rangle$ 之间的跃迁由拉比频率为 Ω_{e2} 、失谐量为 Δ 的非共振激光驱动, 如图1所示.

在旋转框架下并取旋转波近似, 系统的哈密顿量可以写成

$$H_I(t) = \Omega_{e0}|e\rangle\langle 0| + \Omega_{e1}|e\rangle\langle 1| + \Omega_{e2}e^{i\Delta t}|e\rangle\langle 2| + \text{H. c.} \quad (3)$$

当大失谐条件

$$\Delta \gg \Omega_{e2} \quad (4)$$

成立时, 使用文献^[56]的有效哈密顿量理论, 可以把方程(3)约化为

$$H_I' = (\Omega_{e0}|e\rangle\langle 0| + \Omega_{e1}|e\rangle\langle 1| + \text{H. c.}) + \frac{\Omega_{e2}^2}{\Delta}(|e\rangle\langle e| - |2\rangle\langle 2|), \quad (5)$$

其中, $\frac{\Omega_{e2}^2}{\Delta}(|e\rangle\langle e| - |2\rangle\langle 2|)$ 是由非共振激光产生的斯塔克能移项. 式(5)中的哈密顿量与原始一步方案^[21]中的哈密顿量相比, 斯塔克能移 $\frac{\Omega_{e2}^2}{\Delta}$ 代替了原方案中的失谐量. 选取 $S_1^L = \text{Span}\{|0\rangle, |1\rangle\}$ 为单比特量子门编码空间, 因此可忽略对计算空间不引起任何效果的 $\frac{\Omega_{e2}^2}{\Delta}|2\rangle\langle 2|$

图1 (网络版彩色)实现单比特非绝热和乐门的原子能级结构和激光控制示意图

Figure 1 (Color online) The atomic level structure and laser control diagram for the implementation of the one-qubit nonadiabatic holonomic gate

一项.

为了获得绕任意轴旋转任意角的单比特旋转门, 将激光拉比频率设定为

$$\begin{aligned}\Omega_{e0} &= -\Omega \cos\phi \sin\frac{\theta}{2} e^{i\varphi}, \\ \Omega_{e1} &= \Omega \cos\phi \cos\frac{\theta}{2}, \\ \frac{\Omega_{e2}^2}{\Delta} &= -2\Omega \sin\phi,\end{aligned}\quad (6)$$

其中 θ 、 ϕ 和 φ 是与时间无关的参数, 用来描述所实现的单比特门的转动轴和转动角. 利用有效拉比频率 $\Omega = \sqrt{|\Omega_{e0}|^2 + \Omega_{e1}^2 + \frac{\Omega_{e2}^4}{\Delta^2}}$, 可把式(5)的哈密顿量写为

$$H_1 = \Omega \cos\phi (|e\rangle\langle b| + \text{H.c.}) - 2\Omega \sin\phi |e\rangle\langle e|, \quad (7)$$

其中,

$$|b\rangle = -\sin\frac{\theta}{2} e^{-i\varphi} |0\rangle + \cos\frac{\theta}{2} |1\rangle, \quad (8)$$

与之对应的暗态为

$$|d\rangle = \cos\frac{\theta}{2} |0\rangle + \sin\frac{\theta}{2} e^{i\varphi} |1\rangle. \quad (9)$$

很容易证明, 当选取演化时间 $t \in [0, \tau_1]$ 满足

$$\Omega\tau_1 = \pi \quad (10)$$

时, 计算空间 $S_1^L = \text{Span}\{|0\rangle, |1\rangle\} = \text{Span}\{|b\rangle, |d\rangle\}$ 满足和乐量子计算的周期性演化条件(1). 另一方面, H_1 不含时间, 因此它与演化算符 $U_1(t) = e^{-iH_1 t}$ 对易, 有 $\langle p|U_1(t)^\dagger H_1 U_1(t)|q\rangle = \langle p|H_1|q\rangle = 0$ ($p, q = 0, 1$), 意味着和乐量子计算的平行移动条件(2)也成立, 表明 $t = \tau_1$ 时的演化矩阵是作用在 S_1^L 上的非绝热和乐门. 此时,

$$U_1(\tau_1) = e^{i(\pi + \pi \sin\phi)} (|b\rangle\langle b| + |e\rangle\langle e| + |d\rangle\langle d|. \quad (11)$$

在 $\{|0\rangle, |1\rangle\}$ 基矢下, 演化矩阵 $U_1(\tau_1)$ 表示为

$$U_1(\tau_1) = \begin{pmatrix} \cos^2\frac{\theta}{2} - \sin^2\frac{\theta}{2} e^{i\pi \sin\phi} & \frac{1}{2} \sin\theta e^{-i\varphi} (1 + e^{i\pi \sin\phi}) \\ \frac{1}{2} \sin\theta e^{i\varphi} (1 + e^{i\pi \sin\phi}) & \sin^2\frac{\theta}{2} - \cos^2\frac{\theta}{2} e^{i\pi \sin\phi} \end{pmatrix}, \quad (12)$$

可等价地写为

$$U_1 = e^{i\frac{\gamma}{2}} e^{-i\frac{\gamma}{2} \mathbf{n} \cdot \boldsymbol{\sigma}}, \quad (13)$$

这是一个任意的单比特非绝热和乐门, 其中

$\mathbf{n} = (\sin\theta \cos\varphi, \sin\theta \sin\varphi, \cos\theta)$ 是旋转轴, $\gamma = \pi + \pi \sin\phi$ 是旋转角度, $\boldsymbol{\sigma} = (\sigma_x, \sigma_y, \sigma_z)$ 是泡利算符. 因此, 为了实现绕任意旋转轴、转动任意角的单比特门, 只需要调节激光器的拉比频率和相位, 使其满足式(6), 而不需要改变失谐量 Δ .

2 两比特非绝热和乐门

现在给出两比特非绝热和乐受控门的实现方案. 为了实现两比特纠缠门, 考虑一个原子-腔结构, 把两个四能级原子囚禁在光腔中. 通过耦合强度为 g 、失谐量为 $-\Delta$ 的光腔, 把每个原子的状态 $|e\rangle$ 和 $|1\rangle$ 耦合起来. 对于第一个原子, 状态 $|e\rangle$ 和 $|2\rangle$ 之间的跃迁由拉比频率为 Ω_1 、失谐量为 $-(\Delta + \delta)$ 的非共振激光驱动; 对于第二个原子, 状态 $|e\rangle$ 和 $|1\rangle$ 之间的跃迁由拉比频率为 Ω_2 、失谐量为 $-(\Delta + \delta)$ 的非共振激光驱动, 如图2所示.

在旋转框架下取旋转波近似, 系统的哈密顿量可以写成

$$H_2(t) = \sum_{j=1,2} g a e^{-i\Delta t} |e\rangle_j \langle 1| + \Omega_1 e^{-i(\Delta + \delta)t} |e\rangle_1 \langle 2| + \Omega_2 e^{-i(\Delta + \delta)t} |e\rangle_2 \langle 1| + \text{H.c.}, \quad (14)$$

其中 $a(a^\dagger)$ 是腔模的产生(湮灭)算符. 为了推导有效哈密顿量, 把上述哈密顿量写为

$$H_2(t) = \sum_{n=1}^N \hat{h}_n \exp(-i\omega_n t) + \hat{h}_n^\dagger \exp(i\omega_n t), \quad (15)$$

其中,

$$\begin{aligned}\hat{h}_1 &= g a |e\rangle_1 \langle 1|, \quad \hat{h}_2 = \Omega_1 |e\rangle_1 \langle 2|, \\ \hat{h}_3 &= g a |e\rangle_2 \langle 1|, \quad \hat{h}_4 = \Omega_2 |e\rangle_2 \langle 1|,\end{aligned}\quad (16)$$

图2 (网络版彩色)实现两比特非绝热和乐门的原子-腔系统示意图
Figure 2 (Color online) Configuration of atom-cavity system for the implementation of the two-qubit nonadiabatic holonomic gate

$$\omega_1 = \omega_3 = A, \quad \omega_2 = \omega_4 = A + \delta. \quad (17)$$

当大失谐条件

$$\begin{aligned} A &\gg g \\ A + \delta &\gg \Omega_1, \Omega_2 \end{aligned} \quad (18)$$

成立时, 根据文献[56]的有效哈密顿量理论, 式(15)所对

应的有效哈密顿量具有如下形式:

$$H_2'(t) = \sum_{m,n=1}^N \frac{1}{2} \left(\frac{1}{\omega_m} + \frac{1}{\omega_n} \right) [\hat{h}_m^\dagger, \hat{h}_n] \exp[i(\omega_m - \omega_n)t]. \quad (19)$$

把 $\hat{h}_m$ 和 ω_m 的表达式(16)和(17)代入式(19), 得到

$$\begin{aligned} H_2'(t) = & \left[\frac{g\Omega_1}{2} \left(\frac{1}{A+\delta} + \frac{1}{A} \right) e^{i\delta t} a |2\rangle_1 \langle 1| + \frac{g\Omega_2}{2} \left(\frac{1}{A+\delta} + \frac{1}{A} \right) e^{i\delta t} a (|1\rangle_2 \langle 1| - |e\rangle_2 \langle e|) + \text{H.c.} \right] \\ & + \sum_{j=1,2} \frac{g^2}{A} (a^\dagger a |1\rangle_j \langle 1| - a a^\dagger |e\rangle_j \langle e|) + \frac{\Omega_1^2}{A+\delta} (|2\rangle_1 \langle 2| - |e\rangle_1 \langle e|) + \frac{\Omega_2^2}{A+\delta} (|1\rangle_2 \langle 1| - |e\rangle_2 \langle e|) \\ & - \frac{g^2}{A} (|1e\rangle_{12} \langle e1| + \text{H.c.}). \end{aligned} \quad (20)$$

为了获得哈密顿量中对量子门编码空间有实质影响的各项, 进一步将上面的哈密顿量写成如下形式:

$$\begin{aligned} H_2'(t) = & \sum_{n=1}^N \hat{h}_n' \exp(-i\omega_n' t) + (\hat{h}_n')^\dagger \exp(i\omega_n' t) \\ & + \sum_{j=1,2} \frac{g^2}{A} (a^\dagger a |1\rangle_j \langle 1| - a a^\dagger |e\rangle_j \langle e|) + \frac{\Omega_1^2}{A+\delta} (|2\rangle_1 \langle 2| - |e\rangle_1 \langle e|) + \frac{\Omega_2^2}{A+\delta} (|1\rangle_2 \langle 1| - |e\rangle_2 \langle e|) \\ & - \frac{g^2}{A} (|1e\rangle_{12} \langle e1| + \text{H.c.}), \end{aligned} \quad (21)$$

其中,

$$\begin{aligned} \hat{h}_1' &= \frac{g\Omega_1}{2} \left(\frac{1}{A+\delta} + \frac{1}{A} \right) a^\dagger |1\rangle_1 \langle 2|, \\ \hat{h}_2' &= \frac{g\Omega_2}{2} \left(\frac{1}{A+\delta} + \frac{1}{A} \right) a^\dagger |1\rangle_2 \langle 1|, \\ \hat{h}_3' &= -\frac{g\Omega_2}{2} \left(\frac{1}{A+\delta} + \frac{1}{A} \right) a^\dagger |e\rangle_2 \langle e|, \end{aligned} \quad (22)$$

$$\omega_1' = \omega_2' = \omega_3' = \delta. \quad (23)$$

当腔场和激光参数满足关系式

$$\delta \gg \frac{g\Omega_j}{2} \left(\frac{1}{A+\delta} + \frac{1}{A} \right), \quad j = 1, 2, \quad (24)$$

时, 可再次应用有效哈密顿量理论^[56], 经过类似式(15)~(20)的推导, 得到

$$\begin{aligned} H_{\text{eff}} = & \frac{g^2\Omega_1\Omega_2}{4\delta} \left(\frac{1}{A+\delta} + \frac{1}{A} \right)^2 |21\rangle_{12} \langle 11| + \text{H.c.} \\ & + \left[\left(\frac{\Omega_1^2}{A+\delta} + \frac{g^2\Omega_1^2}{4\delta} \left(\frac{1}{A+\delta} + \frac{1}{A} \right)^2 \right) |2\rangle_1 \langle 2| + \left(\frac{\Omega_2^2}{A+\delta} + \frac{g^2\Omega_2^2}{4\delta} \left(\frac{1}{A+\delta} + \frac{1}{A} \right)^2 \right) |1\rangle_2 \langle 1| \right. \\ & - \frac{g^2\Omega_1^2}{4\delta} \left(\frac{1}{A+\delta} + \frac{1}{A} \right)^2 a^\dagger a (|2\rangle_1 \langle 2| + |1\rangle_1 \langle 1|) + \sum_{j=1,2} \frac{g^2}{A} a^\dagger a |1\rangle_j \langle 1| \\ & \left. - \sum_{j=1,2} \left(\frac{g^2}{A} a a^\dagger |e\rangle_j \langle e| + \frac{\Omega_j^2}{A+\delta} |e\rangle_j \langle e| \right) - \frac{g^2\Omega_2^2}{4\delta} \left(\frac{1}{A+\delta} + \frac{1}{A} \right)^2 |e\rangle_2 \langle e| \right] - \frac{g^2}{A} (|1e\rangle_{12} \langle e1| + \text{H.c.}), \end{aligned} \quad (25)$$

其中, $|ij\rangle = |i\rangle_1 \otimes |j\rangle_2$, 括号 $\{\dots\}$ 中的各部分是斯塔克能移项. 式(25)中第二行的斯塔克能移部分可以施加辅助激光来消除^[57], 第三行中的斯塔克能移项在光腔初始

的震荡模式是真空态时自然为0, 不需要施加辅助激光来消除; 而第四行中的所有项作用到计算空间 $S_2^L = \text{Span}\{|00\rangle, |01\rangle, |10\rangle, |11\rangle\}$ 时不会产生任何效果,

因此可以略去, 得到最终的有效哈密顿量为

$$H_{\text{eff}} = \Omega_{12}|21\rangle_{12}\langle 11| + \text{H.c.}, \quad (26)$$

其中,

$$\Omega_{12} = \frac{g^2 \Omega_1 \Omega_2}{4\delta} \left(\frac{1}{A+\delta} + \frac{1}{A} \right)^2. \quad (27)$$

基于式(26)的有效哈密顿量表达式, 可以计算出演化算符 $U_2(t) = e^{-iH_{\text{eff}}t}$, 它在计算基矢 $|00\rangle, |01\rangle, |10\rangle, |11\rangle$ 及其辅助基矢 $|21\rangle$ 下的表示为

$$U_2(t) = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & \cos\Omega_{12}t & -i\sin\Omega_{12}t \\ 0 & 0 & 0 & -i\sin\Omega_{12}t & \cos\Omega_{12}t \end{pmatrix}. \quad (28)$$

当演化时间 $t \in [0, \tau_2]$ 满足

$$\Omega_{12}\tau_2 = \pi \quad (29)$$

时, 演化算符 $U_2(\tau_2) = |00\rangle\langle 00| + |01\rangle\langle 01| + |10\rangle\langle 10| - |11\rangle\langle 11| - |21\rangle\langle 21|$, 意味着初始时刻从计算空间 $S_2^L = \text{Span}\{|00\rangle, |01\rangle, |10\rangle, |11\rangle\}$ 出发的任何量子态, 经过 τ_2 时间的演化后又回到了这个子空间, 即子空间 S_2^L 满足循环演化条件式(1). 利用对易关系 $[H_{\text{eff}}, U_2(t)] = 0$, 可以证明 $\langle \mu | U_2(t)^\dagger H_{\text{eff}} U_2(t) | \nu \rangle = \langle \mu | H_{\text{eff}} | \nu \rangle = 0$ ($\mu, \nu = 00, 01, 10, 11$), 即平行移动条件式(2)也成立. 因此, 在计算空间内 $U_2(\tau_2)$ 扮演着一个两比特非绝热和乐量子门的角色. 在计算空间内演化矩阵 $U_2(\tau_2)$ 的作用等价于两比特受控门:

$$U_2 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}. \quad (30)$$

至此, 我们已经给出了基于原子-腔系统实现两比特非绝热和乐受控门的方案, 它与前面的非绝热单比特门一起组成了一组通用量子门.

3 总结和讨论

我们提出了基于原子-腔系统实现非绝热和乐量子计算的方案, 给出了以四能级原子的基态为编码空间的任意单比特非绝热和乐门和两比特非绝热和乐受控相位门. 与已有的基于原子-腔系统的非绝热和乐量子计算方案相比, 本文的主要进步体现在两比特受控门的设计上, 即利用腔模耦合两个四能级原子, 在激光脉冲的操控下, 通过公共腔模交换虚光子实现双原子基态和辅助状态之间的跃迁, 同时把有效的双原子跃迁从公共的腔模中分离出来, 使得两比特受控门对腔模的衰减不敏感. 在任意单比特门的设计上, 与已有的非绝热和乐量子计算的一步方案相比, 我们利用非共振激光产生的斯塔克能移替代了已有方案中的失谐量, 使得仅通过改变激光器的拉比频率和相位就可以一步实现任意的单比特非绝热和乐门.

在我们的设计方案中, 没有考虑能级自发辐射和腔场衰减的影响, 使得在大失谐近似下单比特门的有效哈密顿量表达式(5)中能够消除能级 $|2\rangle$ 的影响, 双比特门的有效哈密顿量表达式(25)中能够消除能级 $|e\rangle$ 和光场衰减的影响, 但是在原始哈密顿量表达式(3)和(14)中是包含这些能级和光场影响的, 为此我们做了数值模拟, 以检验在自发辐射和腔场衰减存在时方案的有效性. 在单比特门中, 以 $\gamma = \pi/2$ 的相位门为例, 选取拉比频率 $\Omega_{e0} = 0$ MHz、 $\Omega_{e1} \approx 2\pi \times 108.25$ MHz、 $\Omega_{e2} = 2\pi \times 750$ MHz、失谐量 $A = 6\Omega_{e2}$ 、能级 $|2\rangle$ 的自发辐射率 $\Gamma = 0.003\Omega_{e2}$ 等符合实验条件的参数值, 取初始态为 $(|0\rangle + |1\rangle)/\sqrt{2}$, 数值计算表明单比特相位门的保真度为99.89%. 在两比特门中, 选取拉比频率 $\Omega_1 = \Omega_2 = 2\pi \times 750$ MHz、腔模的耦合强度 $g = \Omega_1$ 、失谐量 $A = 18\Omega_1$ 、 $\delta = \Omega_1$ 、能级 $|e\rangle$ 的自发辐射率 $\Gamma = 0.003\Omega_1$, 取初始态为 $|11\rangle$, 数值计算表明两比特受控门的保真度为98.70%. 我们也计算了光场衰减对两比特门的影响, 选取光场的衰减率 $\kappa = 0.003\Omega_1$, 并选取与上述同样的拉比频率、腔模的耦合强度、失谐量以及初始态, 得到量子门的保真度为99.25%.

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Summary for “基于原子-腔系统的非绝热和乐量子计算”

Nonadiabatic holonomic quantum computation with atom-cavity system

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Quantum computation is much more effective than classical computation in solving many problems such as factoring large integers and searching unsorted databases. However, such effectiveness relies on the ability to perform universal high-fidelity quantum gates. One main challenge in achieving such high-fidelity gates is to reduce control errors of a quantum system. To overcome this problem, various proposals of fault-tolerant quantum computation are proposed. A promising one of such proposals is nonadiabatic holonomic quantum computation.

Nonadiabatic holonomic quantum computation is realized by using a quantum system with a subspace satisfying both the cyclic evolution and parallel transport conditions. For an N dimensional quantum system with Hamiltonian $H(t)$ and evolution operator $U(t) = T\exp[-i\int_0^t H(t')dt']$, if there exists a time-dependent L dimensional subspace $S(t)$ spanned by the orthonormal vectors $\{|\phi_k(t)\rangle = U(t)|\phi_k(0)\rangle\}_{k=1}^L$ that satisfy the two conditions: (1) $\sum_{k=1}^L |\phi_k(\tau)\rangle\langle\phi_k(\tau)| = \sum_{k=1}^L |\phi_k(0)\rangle\langle\phi_k(0)|$, with τ being the evolution period, and (2) $\langle\phi_k(t)|H(t)|\phi_l(t)\rangle = 0, k, l = 1, 2, \dots, L$, then the unitary transformation $U(\tau)$ is a holonomic gate on the L dimensional subspace $S(0)$ spanned by $\{|\phi_k(0)\rangle\}_{k=1}^L$. The work of realizing nonadiabatic holonomic gate is to build a physical system with a Hamiltonian satisfying both the cyclic evolution condition and the parallel transport condition.

Nonadiabatic holonomic quantum computation is based on nonadiabatic non-Abelian geometric phases. Since nonadiabatic non-Abelian geometric phases are only dependent on evolution paths but independent of evolution details, nonadiabatic holonomic gates are robust against control errors. Besides, nonadiabatic holonomic gates do not require the long run-time evolution that is necessary for adiabatic holonomic gates. Due to the merits of both robustness against control errors and high-speed realization, nonadiabatic holonomic quantum computation has received increasing attention in theory and experiment. Various schemes of nonadiabatic holonomic quantum computation have been proposed based on different physical systems, and a number of these schemes have been experimentally demonstrated with nuclear magnetic resonance systems, nitrogen-vacancy center systems, and superconducting systems. The progress of nonadiabatic holonomic quantum computation is also reflected in the improvement of the methods of designing quantum gates. The single-shot scheme, single-loop multiple-pulse scheme and composite gate scheme of nonadiabatic quantum gate are based on the improved methods. In particular, the single-shot scheme simplifies the operation of the one-qubit gate, so that the operation of any one-qubit gate, which needs to be realized by combining two non-commuting π -rotation gate in the original scheme, can be simply realized by a single-shot operation.

In this paper, we propose a scheme of nonadiabatic holonomic quantum computation with atom-cavity system. The universal set of quantum gates in our scheme consists of an arbitrary one-qubit nonadiabatic holonomic gate and a two-qubit nonadiabatic holonomic controlled-PHASE gate. To realize the arbitrary one-qubit gate, we use two resonant laser pulses and an off-resonant laser pulse to drive a single four-level atom. With the aid of Stark shifts, the one-qubit gate can be performed in one-step only by changing the Rabi frequencies and the phases of the laser pulses. To realize the two-qubit controlled-PHASE gate, we use two off-resonant laser pulses to drive the two four-level atoms in the cavity. The transitions between the two-atom ground state and the two-atom auxiliary state is facilitated by exchanging virtual photons through a common cavity mode. In this process, the effective two-atom transition is disentangled from the cavity mode and therefore the two-qubit gate is insensitive to the cavity decay.

quantum computation, geometric phases, holonomic quantum computation, atom-cavity system

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