基于真9-qubit纠缠态实现受控双向量子隐形传态

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收稿日期: 2018-10-08; 接受日期: 2018-11-30; 网络出版日期: 2019-03-26

广东省自然科学基金(批准号: 2016a030313736)资助项目

摘要 最近, 基于多方纠缠态的双向受控隐形传态受到广泛关注, 多个基于五方纠缠态、六方纠缠态、七方纠缠态的双向受控隐形传态方案相继被提出. 本文研究了真9-qubit纠缠态对于双向受控隐形传态的有用性, 提出了两个基于真9-qubit纠缠态的双向受控隐形传态方案: 一个方案实现一方传输未知的任意单qubit态而另一方传输未知的任意3-qubit态; 另一个方案实现双方分别传输一个未知的任意2-qubit态. 与其他基于多方纠缠态的方案相比, 我们的方案其信道容量更大, 一次能够双向共传输4个任意的qubit态, 其通信效率也更高, 而且使用的真9-qubit纠缠态易于通过Hadamard门以及受控非门等简单的量子门电路实现.

关键词 双向量子隐形传态, 受控量子隐形传态, 真9-qubit纠缠态, 真多方纠缠

PACS: 03.67.-a, 03.67.HK, 03.67.mn

1 引言

量子隐形传态^[1]是量子通信领域一个重要的研究方向, 它是指通信双方利用事先共享的纠缠态作为信道, 实现将一个未知的量子态瞬间从甲方传输到乙方, 并同时在甲方立即消失.

1993年, Bennett等人^[2]提出了第一个量子隐形传态(Quantum Teleportation, QT)的通信协议. 在Bennett的协议里, Alice和Bob事先共享了一对EPR对作为量子信道, Alice首先对她拥有的EPR对中的qubit和待传输的qubit使用Bell基进行联合测量, 然后通过经典信道将测量结果发送给Bob; Bob收到Alice的测量结果

后, 选择合适的Pauli算子对他拥有的EPR对中的那个粒子进行酉运算就可以恢复出Alice待传输的未知qubit的状态. 1998年, Karlsson和Bourennane^[3]提出了一个受控量子隐形传态方案(Controlled Quantum Teleportation, CQT), 它是利用GHZ态来实现的, 即: Alice, Bob和Charlie事先共享一个三方GHZ态(各持有该GHZ态的一个粒子), 在第三方Charlie的控制下, Alice可以将1个未知的qubit态隐形传输给Bob.

众所周知, 纠缠是一种重要的信息资源, 它在量子隐形传态及其他量子通信领域中都起着关键作用, 如量子超密度编码^[4]、量子密钥分发^[5]、量子秘密共享^[6]、量子安全直接通信^[7-10]等. 最常见的纠缠态是

引用格式: 龙银香, 邵忠良. 基于真9-qubit纠缠态实现受控双向量子隐形传态. 中国科学: 物理学 力学 天文学, 2019, 49: 070301

Long Y X, Shao Z L. Bidirectional controlled quantum teleportation by a genuine entangled 9-qubit state (in Chinese). Sci Sin-Phys Mech Astron, 2019, 49: 070301, doi: 10.1360/SSPMA2018-00341

Bell态、GHZ态、W态以及d级的Bell态、d级的GHZ态等, 学者们研究了各种各样的纠缠态作为量子信道的隐形传态方案, 例如: (1) 提出了基于各种Bell态、GHZ态、W态及其积态作为量子信道的隐形传态方案^[11-19]; (2) 提出了基于各种d级Bell态、d级GHZ态及其积态的量子隐形传态方案^[20-23]; (3) 提出了基于各种真多方纠缠态(如四方Cluster态、五方Cluster态、六方Cluster态、八方Cluster态、真多方纠缠态、最大真六方纠缠态、最大真七方纠缠态等)受控的或非受控的隐形传态协议^[24-34].

2013年, Zha等人^[35]提出了一个双向受控量子隐形传态(Bidirectional Controlled Quantum Teleportation, BCQT)的协议, 即信息的发送者同时还是信息的接受者, Alice和Bob可以在第三方Charlie的监督下相互隐形传输未知的量子态. 之后, 双向受控量子隐形传态受到广泛地关注, 学者们提出了基于各种多方纠缠态的双向受控量子隐形传态的理论方案, 如, (1) 基于各种Cluster态: 五方Cluster态^[35]、六方Cluster态^[36,37]、七方Cluster态^[38]; (2) 基于最大多方纠缠态: 最大六方纠缠态^[39]、最大七方纠缠态^[40]; (3) 基于真多方纠缠态: 真五方纠缠态^[41]、真六方纠缠态^[42]、真七方纠缠态^[43,44]; (4) qudit纠缠态^[45]. 另外, 也有一些方案是没有监督者的双向量子隐形传态(Bidirectional Quantum Teleportation, BQT), 例如: Hassanpour和Houshmand^[46], Sang^[47]和Zadeh等人^[48]分别给出了基于GHZ态、基于五方Cluster态、基于真八方纠缠态的BQT协议.

本文提出了两个新的双向受控量子隐形传态的方案, 它们基于一个真9-qubit纠缠态, 可以实现在Charlie的控制下, Alice和Bob分别互相隐形传输一个任意的2-qubit态, 也可以一方隐形传输一个任意的qubit态而另一方隐形传输一个任意的3-qubit态, 因此两种方案总共都可以隐形传输4个任意的未知qubit态. 下面第2节分两个小节分别介绍这两个隐形传态方案, 第3节进行总结.

2 利用真9-qubit纠缠态进行受控双向隐形传态的方案

2.1 非对称隐形传态的方案

在本节, 我们将介绍一个利用真9-qubit纠缠态进行双向非对称受控隐形传态的方案: 在第三方的控制

下, 一方传输一个未知的任意单qubit态, 另一方传输一个未知的任意3-qubit态.

首先, 我们给出本文中要用到的真9-qubit纠缠态的形式:

$$\begin{aligned} |\varphi\rangle_{123456789} = & \frac{1}{4}(|000000000\rangle + |111100000\rangle \\ & + |110011000\rangle + |110000110\rangle \\ & + |001111000\rangle + |001100110\rangle \\ & + |000011110\rangle + |111111110\rangle \\ & + |110000001\rangle + |001100001\rangle \\ & + |000011001\rangle + |000000111\rangle \\ & + |111111001\rangle + |111100111\rangle \\ & + |110011111\rangle + |001111111\rangle). \end{aligned} \quad (1)$$

这里及以后, 下标{1,2,3,4,5,6,7,8,9}代表粒子的序号.

假设Alice, Bob, Charlie事先共享了一个形如式(1)的9-qubit纠缠态. 其中, Alice拥有序号为1, 3, 5, 7的4个qubit, Bob拥有序号为2, 4, 6, 8的4个qubit, Charlie拥有第9个qubit. 现在, Alice要隐形传输一个未知的单qubit态给Bob:

$$|\phi\rangle_a = a_0|0\rangle + a_1|1\rangle, \quad (2)$$

其中, 参数满足 $|a_0|^2 + |a_1|^2 = 1$. Bob要隐形传输一个未知的3-qubit态给Alice:

$$\begin{aligned} |\mu\rangle_{bcd} = & b_{000}|000\rangle + b_{001}|001\rangle + b_{010}|010\rangle \\ & + b_{011}|011\rangle + b_{100}|100\rangle + b_{101}|101\rangle \\ & + b_{110}|110\rangle + b_{111}|111\rangle, \end{aligned} \quad (3)$$

其中, 参数满足: $\sum_{i,j,k=0}^1 |b_{ijk}|^2 = 1$. Charlie则充当监督者.

设Bell基如下:

$$\begin{aligned} |B_{00}\rangle &= \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle), \\ |B_{01}\rangle &= \frac{1}{\sqrt{2}}(|01\rangle + |10\rangle), \\ |B_{10}\rangle &= \frac{1}{\sqrt{2}}(|00\rangle - |11\rangle), \\ |B_{11}\rangle &= \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle). \end{aligned} \quad (4)$$

设Pauli算子基如下:

$$\begin{aligned}\sigma_{00} &= I = |0\rangle\langle 0| + |1\rangle\langle 1|, \quad \sigma_{01} = X = |1\rangle\langle 0| + |0\rangle\langle 1|, \\ \sigma_{10} &= Z = |0\rangle\langle 0| - |1\rangle\langle 1|, \quad \sigma_{11} = Y = i|1\rangle\langle 0| - i|0\rangle\langle 1|.\end{aligned}\quad (5)$$

注意到上面的9-qubit量子态 $|\varphi\rangle_{123456789}$ 可以写成以下的关于五方割(12|34|56|78|9)的广义Schmidt分解形式^[49]:

$$\begin{aligned}\frac{1}{\sqrt{2}} &(|B_{00}\rangle_{12}|B_{00}\rangle_{34}|B_{00}\rangle_{56}|B_{00}\rangle_{78}|+\rangle_9 \\ &+ |B_{10}\rangle_{12}|B_{10}\rangle_{34}|B_{10}\rangle_{56}|B_{10}\rangle_{78}|-\rangle_9),\end{aligned}\quad (6)$$

其中,

$$|+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle), \quad |-\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle).\quad (7)$$

由此不难知道, 该9-qubit纠缠态具有以下性质: (1) 所有的1-方|8-方割之间具有1 ebit的纠缠; (2) 存在(但不是全部)2-方|7-方割(例如割(13|2456789))之间有2 ebit的纠缠; (3) 存在(但不是全部)3-方|6-方割(例如割(135|246789))之间有3 ebit的纠缠; (4) 存在(但不是全部)4-方|5-方割(例如割(1357|24689))之间有4 ebit的纠缠; (5) 对该9-qubit纠缠态的任意一个qubit进行测量, 其余的qubits仍然保持有纠缠. 也就是说, 该纠缠态是一个真9-qubit纠缠态, 但不是最大9-qubit纠缠态.

$|\varphi\rangle_{123456789}$, $|\phi\rangle_a$ 和 $|\mu\rangle_{bcd}$ 的复合系统为 $|\varphi\rangle_{123456789}|\phi\rangle_a|\mu\rangle_{bcd}$, 记为 $|\zeta\rangle_{123456789abcd}$, 简记为 $|\zeta\rangle$. 则有

$$\begin{aligned}|\zeta\rangle &= \frac{1}{\sqrt{2}}|B_{00}\rangle_{12}|B_{00}\rangle_{34}|B_{00}\rangle_{56}|B_{00}\rangle_{78}|+\rangle_9(a|0\rangle \\ &+ a|1\rangle)_a \otimes (b_{000}|000\rangle + b_{001}|001\rangle \\ &+ b_{010}|010\rangle + b_{011}|011\rangle + b_{100}|100\rangle \\ &+ b_{101}|101\rangle + b_{110}|110\rangle + b_{111}|111\rangle)_{bcd} \\ &+ \frac{1}{\sqrt{2}}|B_{10}\rangle_{12}|B_{10}\rangle_{34}|B_{10}\rangle_{56}|B_{10}\rangle_{78}|-\rangle_9(a|0\rangle \\ &+ a|1\rangle)_a \otimes (b_{000}|000\rangle + b_{001}|001\rangle + b_{010}|010\rangle \\ &+ b_{011}|011\rangle + b_{100}|100\rangle + b_{101}|101\rangle + b_{110}|110\rangle \\ &+ b_{111}|111\rangle)_{bcd}.\end{aligned}$$

为方便表示, 将上式表示为

$$\begin{aligned}|\zeta\rangle &= \frac{1}{\sqrt{2}}|\tau\rangle_{12a}|\nu\rangle_{345678bcd}|+\rangle_9 \\ &+ \frac{1}{\sqrt{2}}|\tau\rangle_{12a}|\nu\rangle_{345678bcd}|-\rangle_9,\end{aligned}\quad (8)$$

其中,

$$\begin{aligned}|\tau\rangle_{12a} &= |B_{00}\rangle_{12}(a|0\rangle + a|1\rangle)_a \\ &= \frac{1}{2}|B_{00}\rangle_{1a}(a|0\rangle + a|1\rangle)_2 + \frac{1}{2}|B_{01}\rangle_{1a} \\ &\quad \otimes (a|0\rangle + a|1\rangle)_2 + \frac{1}{2}|B_{10}\rangle_{1a}(a|0\rangle - a|1\rangle)_2 \\ &\quad + \frac{1}{2}|B_{11}\rangle_{1a}(a|0\rangle - a|1\rangle)_2 \\ &= \sum_{i,j=0}^1 \frac{1}{2}(\sigma_{i,j}|B_{00}\rangle_{1a}) \\ &\quad \otimes [\sigma_{i,j}(a|0\rangle + a|1\rangle)_2],\end{aligned}\quad (9)$$

$$\begin{aligned}|\tau\rangle_{12a} &= |B_{10}\rangle_{12}(a|0\rangle + a|1\rangle)_a \\ &= \frac{1}{2}|B_{00}\rangle_{1a}(a|0\rangle - a|1\rangle)_2 + \frac{1}{2}|B_{01}\rangle_{1a} \\ &\quad \otimes (a|0\rangle + a|1\rangle)_2 + \frac{1}{2}|B_{10}\rangle_{1a} \\ &\quad \otimes (a|0\rangle + a|1\rangle)_2 \\ &\quad + \frac{1}{2}|B_{11}\rangle_{1a}(a|0\rangle + a|1\rangle)_2 \\ &= \sum_{i,j=0}^1 \frac{1}{2}(\sigma_{i,j}|B_{00}\rangle_{1a}) \\ &\quad \otimes [\sigma_{10}\sigma_{i,j}(a|0\rangle + a|1\rangle)_2],\end{aligned}\quad (10)$$

$$\begin{aligned}|\nu\rangle_{345678bcd} &= |B_{00}\rangle_{34}|B_{00}\rangle_{56}|B_{00}\rangle_{78} \sum_{i,j,k=0}^1 b_{ijk}|ijk\rangle_{bcd} \\ &= \sum_{i,j,k,l,m,n=0}^1 \frac{1}{8}[(\sigma_{i,j} \otimes I)|B_{00}\rangle_{4b}] \\ &\quad \otimes [(\sigma_{k,l} \otimes I)|B_{00}\rangle_{6c}][(\sigma_{m,n} \otimes I)|B_{00}\rangle_{8d}] \\ &\quad \otimes [(\sigma_{i,j} \otimes \sigma_{k,l} \otimes \sigma_{m,n}) \\ &\quad \times \sum_{i,j,k=0}^1 b_{ijk}|ijk\rangle_{357}],\end{aligned}\quad (11)$$

$$\begin{aligned}|\nu\rangle_{345678bcd} &= |B_{10}\rangle_{34}|B_{10}\rangle_{56}|B_{10}\rangle_{78} \sum_{i,j,k=0}^1 b_{ijk}|ijk\rangle_{bcd} \\ &= \sum_{i,j,k,l,m,n=0}^1 \frac{1}{8}[(\sigma_{i,j} \otimes I)|B_{00}\rangle_{4b}] \\ &\quad \otimes [(\sigma_{k,l} \otimes I)|B_{00}\rangle_{6c}][(\sigma_{m,n} \otimes I)|B_{00}\rangle_{8d}] \\ &\quad \otimes [(\sigma_{10}\sigma_{i,j}) \otimes (\sigma_{10}\sigma_{k,l}) \otimes (\sigma_{10}\sigma_{m,n}) \\ &\quad \times \sum_{i,j,k=0}^1 b_{ijk}|ijk\rangle_{357}].\end{aligned}\quad (12)$$

综合以上的式(8)–(12)可知:

(1) 如果Charlie对粒子9使用基 $\{|+\rangle, |-\rangle\}$ 进行测量,

Alice对粒子对 $\{1, a\}$ 使用Bell基 $\{\sigma_{ij}|B_{00}\rangle_{1a} |i, j = 0, 1\}$ 进行联合测量, 则测量后粒子2将根据粒子对 $\{1, a\}$ 以及粒子9的测量结果对应地塌缩到以下状态之一:
 $\sigma_{ij}(a_0|0\rangle + a_1|1\rangle)_2$ 或 $\sigma_{10}\sigma_{ij}(a_0|0\rangle + a_1|1\rangle)_2, i, j=0, 1$.

(2) 如果Charlie对粒子9使用基 $\{|+\rangle, |-\rangle\}$ 进行测量, Bob分别对粒子对 $\{4, b\}$, $\{6, c\}$ 和 $\{8, d\}$ 使用Bell基 $\{\sigma_{ij}|B_{00}\rangle_{4b} |i, j = 0, 1\}$, $\{\sigma_{kl}|B_{00}\rangle_{6c} |k, l = 0, 1\}$ 和 $\{\sigma_{mn}|B_{00}\rangle_{8d} |m, n = 0, 1\}$ 进行联合测量, 则测量后粒子对 $\{3, 5, 7\}$ 将根据Bob和Charlie的测量结果对应地塌缩到以下状态之一:

$$\left[(\sigma_{ij} \otimes \sigma_{kl} \otimes \sigma_{mn}) \sum_{i,j,k=0}^1 b_{ijk} |ijk\rangle_{357} \right]$$

或 $(\sigma_{10}\sigma_{ij}) \otimes (\sigma_{10}\sigma_{kl}) \otimes (\sigma_{10}\sigma_{mn}) \sum_{i,j,k=0}^1 b_{ijk} |ijk\rangle_{357}, i, j, k, l, m, n=0, 1$.

因此, 我们可以构造双向隐形传态协议如下:

协议1 一方传输单qubit, 另一方传输3-qubit.

Step 1 信道准备阶段. 在该阶段, 我们要制备纠缠态^[50,51]. 设9-qubit初始态为 $|000000000\rangle_{123456789}$, 则可以按以下过程制备该真9-qubit纠缠态.

(1) 对序号为1, 3, 5, 7, 9的粒子应用Hadamard算子, 结果9-qubit态演变为

$$(|+\rangle|0\rangle|+\rangle|0\rangle|+\rangle|0\rangle|+\rangle|0\rangle|+\rangle)_{123456789};$$

(2) 对粒子对1和2, 3和4, 5和6, 7和8分别应用受控非门CNOT₁₂, CNOT₃₄, CNOT₅₆, CNOT₇₈, 其中粒子1, 3, 5, 7分别为控制比特. 结果9-qubit演变成

$$|B_{00}\rangle_{12}|B_{00}\rangle_{34}|B_{00}\rangle_{56}|B_{00}\rangle_{78}|+\rangle_9;$$

(3) 分别对粒子9和1、粒子9和3、粒子9和5以及粒子9和7应用下面的受控Z门CZ = $|00\rangle\langle 00| + |01\rangle\langle 01| + |10\rangle\langle 10| - |11\rangle\langle 11|$, 其中粒子9为控制比特. 结果9-qubit演变成

$$\frac{1}{\sqrt{2}}|B_{00}\rangle_{12}|B_{00}\rangle_{34}|B_{00}\rangle_{56}|B_{00}\rangle_{78}|0\rangle_9 + \frac{1}{\sqrt{2}}|B_{10}\rangle_{12}|B_{10}\rangle_{34}|B_{10}\rangle_{56}|B_{10}\rangle_{78}|1\rangle_9;$$

(4) 对粒子9应用Hadamard算子H = $|+\rangle\langle 0| + |-\rangle\langle 1|$. 结果9-qubit演变成我们所需要的真9-qubit纠缠态, 即式(1)的形式, 如图1所示.

接下来, Alice将制备好的9-qubit纠缠态的粒子分发给Bob和Charlie, 其中粒子2, 4, 6和8分发给Bob, 粒子9分发给Charlie, 粒子1, 3, 5和7保留给自己.

Step 2 测量阶段. Alice对待传输的qubit $|\phi\rangle_a$ 和9-qubit纠缠态中的粒子1使用Bell基进行联合测量; Bob对待传输的3-qubit $|\mu\rangle_{bcd}$ 中粒子b和9-qubit纠缠态中的粒子4使用Bell基进行联合测量, 对待传输的3-qubit $|\mu\rangle_{bcd}$ 中粒子c和9-qubit纠缠态中的粒子6使用Bell基进行联合测量, 对待传输的3-qubit $|\mu\rangle_{bcd}$ 中粒子

图 1 制备真9-qubit纠缠态的量子线路

Figure 1 Quantum circuit to generate the genuine entangled 9-qubit state.

d 和9-qubit纠缠态中的粒子8使用Bell基进行联合测量; Charlie对粒子9使用基 $\{|+\rangle, |-\rangle\}$ 进行测量.

Step 3 经典信息传输阶段. Alice将她的测量结果编码成2 bits信息, 并通过经典信道发送给Bob; Bob将他对粒子4和 b 、粒子6和 c 以及粒子8和 d 的测量结果分别编码成2 bits信息, 并将总共6 bits信息通过经典信道传输给Alice; Charlie将他的测量结果编码成1 bit信息, 并通过经典信道广播给Alice和Bob. 编码规则如下:

$$\begin{aligned} |B_{00}\rangle &\rightarrow 00, |B_{01}\rangle \rightarrow 01, |B_{10}\rangle \rightarrow 10, |B_{11}\rangle \rightarrow 11, \\ |+\rangle &\rightarrow 0, |-\rangle \rightarrow 1. \end{aligned}$$

Step 4 重构量子态阶段. Bob根据Charlie的测量结果以及Alice的测量结果, 可以选择一个合适的Pauli算子对他拥有的粒子2进行酉运算即可以在该粒子上恢复出Alice要传输的qubit $|\phi\rangle_a$, 参见表1. Alice根据Charlie的测量结果以及Bob的测量结果, 可以选择3个合适的Pauli算子分别对她拥有的粒子3, 5和7进行酉运算即可以在该粒子组上恢复出Bob要传输的3-qubit量子态 $|\mu\rangle_{bcd}$, 参见表2.

2.2 对称隐形传态方案

接下来, 我们将分析利用前面的真9-qubit纠缠态实现双方各传输一个任意的未知2-qubit态的情况, 这是一个对称隐形传态方案. 我们仍然假设Alice, Bob,

Charlie事先共享了一个如式(1)所示的真9-qubit纠缠态. 其中, Alice拥有序号为1, 3, 5和7的4个qubit, Bob拥有序号为2, 4, 6和8的4个qubit, Charlie拥有第9个qubit. 现在, Alice要隐形传输一个未知的任意2-qubit态给Bob:

$$|\phi\rangle_{ab} = a_{00}|00\rangle + a_{01}|01\rangle + a_{10}|10\rangle + a_{11}|11\rangle, \quad (13)$$

其中, $|a_{00}|^2 + |a_{01}|^2 + |a_{10}|^2 + |a_{11}|^2 = 1$. Bob要同时隐形传输一个未知的任意2-qubit态给Alice:

$$|\mu\rangle_{cd} = b_{00}|00\rangle + b_{01}|01\rangle + b_{10}|10\rangle + b_{11}|11\rangle, \quad (14)$$

其中, $|b_{00}|^2 + |b_{01}|^2 + |b_{10}|^2 + |b_{11}|^2 = 1$. Charlie则充当监督者.

采用和前面相似的思路, 我们首先写出 $|\phi\rangle_{123456789}$,

$|\phi\rangle_{ab}$ 和 $|\mu\rangle_{cd}$ 的复合系统为 $|\phi\rangle_{123456789}|\phi\rangle_{ab}|\mu\rangle_{cd}$, 记为 $|\eta\rangle_{123456789abcd}$, 简记为 $|\eta\rangle$. 则有

$$\begin{aligned} |\eta\rangle &= |B_{00}\rangle_{12}|B_{00}\rangle_{34}|B_{00}\rangle_{56}|B_{00}\rangle_{78}|+\rangle_9 \\ &\quad \otimes (a_{00}|00\rangle + a_{01}|01\rangle + a_{10}|10\rangle + a_{11}|11\rangle)_{ab} \\ &\quad \otimes (b_{00}|00\rangle + b_{01}|01\rangle + b_{10}|10\rangle + b_{11}|11\rangle)_{cd} \\ &\quad + |B_{10}\rangle_{12}|B_{10}\rangle_{34}|B_{10}\rangle_{56}|B_{10}\rangle_{78}|-\rangle_9 \\ &\quad \otimes (a_{00}|00\rangle + a_{01}|01\rangle + a_{10}|10\rangle + a_{11}|11\rangle)_{ab} \\ &\quad \otimes (b_{00}|00\rangle + b_{01}|01\rangle + b_{10}|10\rangle + b_{11}|11\rangle)_{cd}. \end{aligned} \quad (15)$$

表 1 Bob在粒子2上选择合适的Pauli算子以恢复Alice传输的单qubit态($i, j=0, 1$)

Table 1 Bob can restore the quantum state teleported by Alice through applying an appropriate Pauli operator on particle 2 ($i, j=0, 1$)

Charlie对粒子9的测量结果	Alice对粒子对1和 a 的联合测量结果	Bob在粒子2上选择合适的Pauli算子恢复 $ \phi\rangle_a$
$ +\rangle$	$\sigma_{i,j} B_{00}\rangle_{1a}$	$\sigma_{i,j}$
$ -\rangle$	$\sigma_{i,j} B_{00}\rangle_{1a}$	$\sigma_{10}\sigma_{i,j}$

表 2 Alice在粒子3, 5和7上分别应用合适的Pauli算子以恢复Bob传输的3-qubit态($i, j, k, l, m, n=0, 1$)

Table 2 Alice can restore the 3-qubit state teleported by Bob through applying an appropriate Pauli operator on particle 3, 5 and 7, respectively ($i, j, k, l, m, n=0, 1$)

Charlie对粒子9的测量结果	Bob对分别粒子对4和 b , 6和 c 以及8和 d 进行Bell基测量的结果	为恢复Bob传输的状态, Alice在粒子3, 5, 7上分别应用合适的Pauli算子
$ +\rangle$	$\sigma_{i,j} B_{00}\rangle_{4b}, \sigma_{k,l} B_{00}\rangle_{6c}, \sigma_{m,n} B_{00}\rangle_{8d}$	$\sigma_{i,j}, \sigma_{k,l}, \sigma_{m,n}$
$ -\rangle$	$\sigma_{i,j} B_{00}\rangle_{4b}, \sigma_{k,l} B_{00}\rangle_{6c}, \sigma_{m,n} B_{00}\rangle_{8d}$	$\sigma_{10}\sigma_{i,j}, \sigma_{10}\sigma_{k,l}, \sigma_{10}\sigma_{m,n}$

为方便表示, 将上式表示为

$$|\eta\rangle = |\epsilon\rangle_{1234ab} |v\rangle_{5678cd} |+\rangle_9 + |\epsilon'\rangle_{1234ab} |v'\rangle_{5678cd} |-\rangle_9, \quad (16)$$

其中,

$$\begin{aligned} |\epsilon\rangle_{1234ab} &= |B_{00}\rangle_{12} |B_{00}\rangle_{34} (a_{00}|00\rangle + a_{01}|01\rangle \\ &\quad + a_{10}|10\rangle + a_{11}|11\rangle)_{ab} \\ &= \sum_{i,j,k,l=0}^1 \frac{1}{4} [(\sigma_{i,j} \otimes I) |B_{00}\rangle_{1a}] \\ &\quad \otimes [(\sigma_{k,l} \otimes I) |B_{00}\rangle_{3b}] \\ &\quad \otimes [(a_{00}|00\rangle + a_{01}|01\rangle \\ &\quad + a_{10}|10\rangle + a_{11}|11\rangle)_{24}], \end{aligned} \quad (17)$$

$$\begin{aligned} |\epsilon'\rangle_{1234ab} &= |B_{10}\rangle_{12} |B_{10}\rangle_{34} (a_{00}|00\rangle + a_{01}|01\rangle \\ &\quad + a_{10}|10\rangle + a_{11}|11\rangle)_{ab} \\ &= \sum_{i,j,k,l=0}^1 \frac{1}{4} [(\sigma_{i,j} \otimes I) |B_{00}\rangle_{1a}] \\ &\quad \otimes [(\sigma_{k,l} \otimes I) |B_{00}\rangle_{3b}] \\ &\quad \otimes [(\sigma_{10}\sigma_{i,j}) \otimes (\sigma_{10}\sigma_{k,l}) (a_{00}|00\rangle + a_{01}|01\rangle \\ &\quad + a_{10}|10\rangle + a_{11}|11\rangle)_{24}], \end{aligned} \quad (18)$$

$$\begin{aligned} |v\rangle_{5678cd} &= |B_{00}\rangle_{56} |B_{00}\rangle_{78} (b_{00}|00\rangle + b_{01}|01\rangle \\ &\quad + b_{10}|10\rangle + b_{11}|11\rangle)_{cd} \\ &= \sum_{i,j,k,l=0}^1 \frac{1}{4} [(\sigma_{i,j} \otimes I) |B_{00}\rangle_{6c}] \\ &\quad \otimes [(\sigma_{k,l} \otimes I) |B_{00}\rangle_{8d}] \\ &\quad \otimes [(b_{00}|00\rangle + b_{01}|01\rangle \\ &\quad + b_{10}|10\rangle + b_{11}|11\rangle)_{57}], \end{aligned} \quad (19)$$

$$\begin{aligned} |v'\rangle_{5678cd} &= |B_{10}\rangle_{56} |B_{10}\rangle_{78} (b_{00}|00\rangle + b_{01}|01\rangle \\ &\quad + b_{10}|10\rangle + b_{11}|11\rangle)_{cd} \\ &= \sum_{i,j,k,l=0}^1 \frac{1}{4} [(\sigma_{i,j} \otimes I) |B_{00}\rangle_{6c}] \\ &\quad \otimes [(\sigma_{k,l} \otimes I) |B_{00}\rangle_{8d}] [(\sigma_{10}\sigma_{i,j}) \otimes (\sigma_{10}\sigma_{k,l}) \\ &\quad \times (b_{00}|00\rangle + b_{01}|01\rangle + b_{10}|10\rangle + b_{11}|11\rangle)_{57}]. \end{aligned} \quad (20)$$

综合式(16)–(20)可知:

(1) 如果Charlie对粒子9使用基 $\{|+\rangle, |-\rangle\}$ 进行测量,

Alice对粒子对 $\{1, a\}$ 和 $\{3, b\}$ 使用Bell基进行联合测量, 则测量后粒子对 $\{2, 4\}$ 将根据粒子对 $\{1, a\}$, $\{3, b\}$ 以及粒子9的测量结果对应地塌缩到以下状态之一:

$$\left\{ (\sigma_{i,j} \otimes \sigma_{k,l}) (a_{00}|00\rangle + a_{01}|01\rangle + a_{10}|10\rangle + a_{11}|11\rangle)_{24} \mid i, j, k, l = 0, 1 \right\};$$

或以下状态之一:

$$\left\{ (\sigma_{10}\sigma_{i,j}) \otimes (\sigma_{10}\sigma_{k,l}) (a_{00}|00\rangle + a_{01}|01\rangle + a_{10}|10\rangle + a_{11}|11\rangle)_{24} \mid i, j, k, l = 0, 1 \right\}.$$

(2) 如果Charlie对粒子9使用基 $\{|+\rangle, |-\rangle\}$ 进行测量,

Bob对粒子对 $\{6, c\}$ 和 $\{8, d\}$ 使用Bell基进行联合测量, 则测量后粒子对 $\{5, 7\}$ 将根据粒子对 $\{6, c\}$, $\{8, d\}$ 以及粒子9的测量结果对应地塌缩到以下状态之一:

$$\left\{ (\sigma_{i,j} \otimes \sigma_{k,l}) (b_{00}|00\rangle + b_{01}|01\rangle + b_{10}|10\rangle + b_{11}|11\rangle)_{57} \mid i, j, k, l = 0, 1 \right\};$$

或以下状态之一:

$$\left\{ (\sigma_{10}\sigma_{i,j}) \otimes (\sigma_{10}\sigma_{k,l}) (b_{00}|00\rangle + b_{01}|01\rangle + b_{10}|10\rangle + b_{11}|11\rangle)_{57} \mid i, j, k, l = 0, 1 \right\}.$$

据此, 采用和2.1节类似的方法我们可以构造出双方各传输两个qubit的受控隐形传态协议. 协议的详细过程省略, 只对该协议作一些说明. 在这个协议里: 第一步状态的制备和协议1相同; 第二步测量阶段, Alice要使用Bell基分别对粒子对 $\{1, a\}$ 和 $\{3, b\}$ 进行联合测量, Bob要使用Bell基分别对粒子对 $\{6, c\}$ 和 $\{8, d\}$ 进行联合测量, Charlie采用和协议1相同的测量方案; 第三步经典信息传输阶段中, Alice要对 $\{1, a\}$ 和 $\{3, b\}$ 的测量结果分别编码成2 bits的信息, Bob粒子对 $\{6, c\}$ 和 $\{8, d\}$ 的测量结果分别编码成2 bits的信息, Charlie将他的测量结果编码成1 bit信息, 编码方法同协议1; 第四步重构量子态阶段, Bob根据Charlie的测量结果以及Alice的测量结果, 可以选择两个合适的Pauli算子分别对他拥有的粒子2和4进行酉运算即可以在该粒子上恢复出Alice要传输的 $|\varphi\rangle_{ab}$, 参见表3. Alice根据Charlie

表 3 Bob在粒子2和4上分别应用合适的Pauli算子以恢复Alice传输的2-qubit态**Table 3** Bob can restore the 2-qubit state teleported by Alice through applying an appropriate Pauli operator on particle 2 and 4, respectively

Charlie对粒子9的测量结果	Alice分别对粒子对 $\{1, a\}$ 和 $\{3, b\}$ 使用Bell基的联合测量结果($i, j, k, l=0, 1$)	为恢复 $ \varphi\rangle_{ab}$, Bob在粒子2和4上运用合适的Pauli算子
$ +\rangle$	$(\sigma_{i,j} \otimes I) B_{00}\rangle_{1a}, (\sigma_{k,l} \otimes I) B_{00}\rangle_{3b}$	$\sigma_{i,j} \otimes \sigma_{k,l}$
$ -\rangle$	$(\sigma_{i,j} \otimes I) B_{00}\rangle_{1a}, (\sigma_{k,l} \otimes I) B_{00}\rangle_{3b}$	$(\sigma_{10}\sigma_{i,j}) \otimes (\sigma_{10}\sigma_{k,l})$

的测量结果以及Bob的测量结果, 可以选择两个合适的Pauli算子分别对她拥有的粒子5和7进行酉运算即可以在该粒子对上恢复出Bob要传输的2-qubit量子态 $|\mu\rangle_{cd}$, 参见表4.

3 结论

本文利用一个真9-qubit纠缠态构造了两个双向受控量子隐形传态的理论方案: (1) 一个方案是一方隐形

传输单qubit态而另一方同时隐形传输3-qubit态; (2) 另一个方案是两方同时各隐形传输一个2-qubit态. 两个方案都是受控传输, 即需要第三方Charlie的监督. 类似于Li等人^[52]的方法, 将我们的方案与现有的其他BCQT的主要方案从以下5个方面进行比较: 消耗的量子资源、消耗的经典资源、传输的量子比特数、通讯的效率以及采用的测量和运算, 结果见表5. 通信效率采用Yuan等人^[53]的定义, 即 $\eta=q_s/(q_u+q_c)$, 其中 η 代表通信效率, q_s 代表传输的qubit数, q_u 代表消耗的qubit数, q_c

表 4 Alice在粒子5和7上分别应用合适的Pauli算子以恢复Bob传输的2-qubit态**Table 4** Alice can restore the 2-qubit state teleported by Bob through applying an appropriate Pauli operator on particle 5 and 7, respectively

Charlie对粒子9的测量结果	Bob分别对粒子对 $\{6, c\}$ 和 $\{8, d\}$ 使用Bell基的联合测量结果($i, j, k, l=0, 1$)	为恢复 $ \mu\rangle_{cd}$, Alice在粒子5和7上运用合适的Pauli算子
$ +\rangle$	$(\sigma_{i,j} \otimes I) B_{00}\rangle_{6c}, (\sigma_{k,l} \otimes I) B_{00}\rangle_{8d}$	$\sigma_{i,j} \otimes \sigma_{k,l}$
$ -\rangle$	$(\sigma_{i,j} \otimes I) B_{00}\rangle_{6c}, (\sigma_{k,l} \otimes I) B_{00}\rangle_{8d}$	$(\sigma_{10}\sigma_{i,j}) \otimes (\sigma_{10}\sigma_{k,l})$

表 5 主要方案的比较(SM: 单qubit测量; BM: Bell基测量; CNOT: 受控非门运算; SU: 单qubit酉运算; CZ: 受控Z门)**Table 5** Comparison between our scheme and the other schemes (SM represents single-qubit measure; BM represents Bell base measure; CNOT represents controlled NOT gate; SU represents unitary operation on single-qubit; CZ represents controlled Z gate)

方案	消耗的量子资源 (q_u)	采用的测量和运算 (q_m, q_o)	消耗的经典bit数 (q_c)	传输的qubit数 (q_s)	通信效率 (η)
文献[35]	5-qubit簇态	5次SM, 2次CNOT, 2次SU	5	2	1/5
文献[36]	6-qubit簇态	6次SM, 1次CZ, 2次SU	6	2	1/6
文献[37]	6-qubit簇态	2次BM, 2次SM, 1次CZ, 2次SU	6	3	1/4
文献[38]	7-qubit簇态	3次BM, 1次SM, 3次SU	7	3	3/14
文献[39]	6-qubit最大纠缠态	2次BM, 2次SM, 1次CZ	6	2	1/6
文献[40]	7-qubit最大纠缠态	2次BM, 3次SM, 2次SU	7	2	1/7
文献[41]	5-qubit纠缠态	2次BM, 1次SM, 2次SU	5	2	1/5
文献[42]	6-qubit真纠缠态	2次BM, 2次SM, 2次SU	6	2	1/6
文献[43]	7-qubit纠缠态	3次BM, 1次SM, 3次SU	7	3	3/14
文献[44]	7-qubit纠缠态	3次BM, 1次SM, 3次SU	7	3	3/14
本文	9-qubit真纠缠态	4次BM, 1次SM, 4次SU	9	4	2/9

代表消耗的bit数.

由表5可知, (1) 与现有的基于多方纠缠态的双向受控隐形传态理论方案相比, 我们的方案其传输容量大, 我们的两个方案都能总共传输4个任意的未知qubit态. (2) 我们的方案的通信效率仅比文献[37]低, 但是应该指出, 文献[37]传输的是一个任意的qubit加一个Schmidt形式的2-qubit. 而Schmidt形式的2-qubit不是任意的qubit, 它只有两个参数, 本质上可以通过传一个qubit再加CNOT运算完成, 考虑到这一点, 文献[37]的真

实通信效率应该只有1/6, 故可以认为本文的方案通信效率最高. (3) 类似于通信效率, 如果我们定义计算效率为 $\eta_c = q_s / (q_m + q_o)$, 其中 η_c 代表计算效率, q_s 代表传输的qubit数, q_m 代表需要的测量次数, q_o 代表需要的运算次数. 则可知我们的方案计算效率也是最高的, 为4/9.

此外, 我们的方案基于的真9-qubit纠缠态比较容易制备, 只需要对初始态 $|000000000\rangle$ 应用一个包含6个Hadamard门、4个受控非门、4个受控Z门的量子线路就可以得到.

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Bidirectional controlled quantum teleportation by a genuine entangled 9-qubit state

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Since the first protocol of quantum teleportation based on an EPR pair was presented by Bennett et al. in 1993, it has become a hot topic in quantum communication. It is known that entanglement plays a key role in quantum teleportation. Various schemes of controlled quantum teleportation and bidirectional controlled quantum teleportation have been developed by using all kinds of multi-parties entangled states as a quantum channel, e.g. GHZ states, W states, 5-qubit cluster states, 6-qubit cluster states, 7-qubit cluster states, maximally entangled 5-qubit states, maximally entangled 6-qubit states, genuine entangled 6-qubit states, genuine entangled 7-qubit states, genuine entangled 8-qubit states. In this paper, a genuine entangled 9-qubit state is used as channel of quantum teleportation, and two schemes of bidirectional controlled quantum teleportation are proposed. One scheme is that with the control of Charlie Alice can transmit an arbitrary unknown qubit state to Bob and Bob can transmit an arbitrary unknown 3-qubit state to Alice in the same time. Another scheme is that with the control of Charlie Alice and Bob can transmit an arbitrary unknown 2-qubit to each other simultaneously. Compared with the existed schemes, our scheme can transmit more qubits of arbitrary state in total by an entangled state, and the intrinsic efficiency of our scheme is maybe the best. Moreover, the genuine entangled 9-qubit state used in our scheme can be produced through some simple quantum unitary gates.

bidirectional quantum teleportation, controlled quantum teleportation, genuine entangled 9-qubit state, genuine multi-parties entanglement

PACS: 03.67.-a, 03.67.HK, 03.67.mn

doi: [10.1360/SSPMA2018-00341](https://doi.org/10.1360/SSPMA2018-00341)