基于干扰观测器的输入饱和多智能体系统事件触发控制

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摘要 针对一类具有未知干扰与输入饱和的严格反馈多智能体系统, 研究其在有向拓扑下的领导跟随一致性问题, 提出一种基于干扰观测器的分布式事件触发控制方法. 该方法首先设计干扰观测器估计未知干扰, 进而结合动态面控制与自适应反步法设计控制输入. 其次, 采用相关阈值事件触发控制策略解决通信资源有限的问题, 并引入饱和补偿系统补偿控制器信号与执行器信号的不匹配. 然后, 通过 Lyapunov 稳定性定理证明闭环系统内所有信号是半全局一致最终有界的. 最后, 仿真结果验证提出控制方法的有效性.

关键词 多智能体系统, 干扰观测器, 输入饱和, 事件触发控制, 动态面控制

1 引言

多智能体系统具有较强的鲁棒性和容错能力等优点, 并且能完成某些单个智能体难以完成的任务. 因此, 在交通控制、海洋探测、机器人、军事等领域有着广泛的应用. 领导跟随一致性问题是多智能体系统协同控制的基本问题之一, 具有重要的实际价值和理论意义, 许多学者对这一问题进行了深入研究并取得了丰硕的成果^[1~7].

工程中, 系统经常工作于复杂甚至是危险的环境中, 各种干扰对系统控制精度有一定的影响, 而多智能体系统领导者与跟随者状态一致对控制精度有严格的要求. 因此, 如何处理系统干扰问题一直是多智能体系统控制设计的关键^[8~16]. 针对包含未知干扰的二阶非线性多智能体系统, Liu 等^[14] 设计了有限时间收敛的非连续干扰观测器. Ding^[15] 针对网络连接的动态系统设计了一种干扰观测器, 实现了抗干扰一致性控制. Li 等^[16] 针对带有不匹配干扰的多智能体系统研究了自抗扰控制. 然而, 当外

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界干扰较大时,抑制干扰往往需要较大的控制输入来补偿,然而多智能体系统受限于机械构造与部件性能等物理因素,易导致输入饱和现象,从而影响控制性能甚至导致系统失稳^[17~23].因此,许多专家对多智能体的输入饱和问题展开了研究.Qin等^[21]和Su等^[22]分别在线性多智能体系统的双边一致性问题中包含控制问题中采用低增益技术处理输入饱和问题.Wang等^[23]结合低-高增益反馈技术和高增益观测器解决了多智能体系统输入饱和问题.

在实际系统中,传统的时间触发控制策略在系统稳定运行时仍然需要高频率采样,这样不仅会造成系统能耗过高,还会导致网络拥挤,而事件触发机制为解决该问题提供了一种有效的策略^[24~31].Hu等^[29]在航天器的编队控制中采用事件触发控制方案以减轻通信负载,节约星载资源.为了避免相邻智能体之间连续通信,Wang等^[30]将事件触发控制策略运用于多智能体系统的差分隐私一致算法.基于二阶非线性多智能体系统的包含控制,Xiang等^[31]研究了集中式和分布式事件触发控制方案.然而,输入饱和与事件触发机制直接约束多智能体系统的控制器信号,若同时考虑干扰为设计控制器设计带来很大的挑战.

鉴于以上分析,本文针对一类带有未知干扰和输入饱和的严格反馈多智能体系统,研究其在事件触发控制下的领导跟随一致性问题.主要工作如下:(1)设计干扰观测器估计未知干扰,补偿干扰对系统的影响.(2)建立饱和模型,采用双曲正切函数近似输入饱和和非线性函数,构造饱和补偿系统补偿控制器信号与执行器信号的不匹配.(3)设计一种基于相对阈值策略的分布式事件触发控制器,缓解智能体通信负担,降低系统能耗.(4)通过仿真结果验证所提出控制方法的有效性.

2 问题阐述及准备

2.1 代数图论

为了便于描述智能体之间的通信关系,引入代数图论的相关知识如下:图 $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{A})$ 表示多智能体系统的有向通信拓扑图,图中每个节点各对应一个智能体.其中, $\mathcal{V} = (1, 2, \dots, N)$ 表示 N 个节点的集合,节点之间边的集合为 $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$,从节点 i 到 j 的边定义为有序对 $(i, j) \in \mathcal{E}$,表示智能体 i 能够接收到智能体 j 的信息,并称节点 i 与 j 相邻.定义 $\mathcal{N}_i = \{j \in \mathcal{V} \mid (i, j) \in \mathcal{E}, i \neq j\}$ 为智能体 i 邻边的集合. $\mathcal{A} = [a_{i,j}] \in \mathbb{R}^{N \times N}$ 表示权重邻接矩阵,如果 $(i, j) \in \mathcal{E}$,那么权重 $a_{i,j} > 0$;否则 $a_{i,j} = 0$.假设拓扑图中不存在自环,即 $a_{i,i} = 0, \forall i \in \mathcal{V}$.节点 i 的入度 $b_i = \sum_{j \in \mathcal{N}_i} a_{i,j}$,定义 $\mathcal{B} = \text{diag}\{b_1, b_2, \dots, b_N\}$ 为入度对角矩阵,则图 $\mathcal{G}$ 的Laplace矩阵为 $\mathcal{L} = \mathcal{B} - \mathcal{A}$.由于多智能体系统中只有部分智能体能直接接收到跟踪轨迹信号,则可将给定跟踪信号当作编号为0的领导者的输出.定义 $\mathcal{C} = \text{diag}\{a_{1,0}, a_{2,0}, \dots, a_{N,0}\}$,同样,如果节点 i 能接收到领导者0的信息,则 $a_{i,0} > 0$;反之, $a_{i,0} = 0$.

引理1 ([32]) 有向图 $\mathcal{G}$ 中,如果存在至少一个节点,能够沿着有向路径抵达其他任一节点,则称有向图 $\mathcal{G}$ 包含生成树,该节点称为生成树的根节点.如果节点0是根节点,那么矩阵 $\mathcal{L} + \mathcal{C}$ 为非奇异.

2.2 问题阐述

根据文献[33],本文研究的多智能体系统中包含 N 个跟随者,其中第 i 个跟随者可描述为一类带有未知参数与未知干扰的 n 阶严格反馈系统

$$\begin{aligned} \dot{x}_{i,m} &= x_{i,m+1} + f_{i,m}(X_{i,m}) + \theta_i^T g_{i,m}(X_{i,m}) + d_{i,m}, \quad m = 1, 2, \dots, n, \\ y_i &= x_{i,1}, \\ u_i &= x_{i,n+1}, \end{aligned} \quad (1)$$

其中, $i = 1, 2, \dots, N$, $X_{i,m} = [x_{i,1}, x_{i,2}, \dots, x_{i,m}]^T \in \mathbb{R}^m$ 表示系统状态, $\theta_i \in \mathbb{R}^r$ 为未知系统参数, r 为正整数, 由系统特性决定, $d_{i,m}$ 表示未知干扰, $f_{i,m}(X_{i,m}) : \mathbb{R}^m \rightarrow \mathbb{R}^1$ 与 $g_{i,m}(X_{i,m}) : \mathbb{R}^m \rightarrow \mathbb{R}^r$ 均为已知的 C^1 非线性函数. 为了便于推导, 下文中将 $f_{i,m}(X_{i,m})$ 与 $g_{i,m}(X_{i,m})$ 分别简写为 $f_{i,m}$ 与 $g_{i,m}$. $y_i \in \mathbb{R}$, $u_i \in \mathbb{R}$ 分别表示系统输出与系统输入.

本文采用基于相对阈值策略的事件触发控制方法, u_i 设定如下:

$$\begin{cases} u_i(t) = v_i(t_{i,k}), & \forall t \in [t_{i,k}, t_{i,k+1}), \\ t_{i,k+1} = \inf\{t > t_{i,k} \mid |\Phi_i| \geq \Delta_i |u_i| + l_i\}, & t_{i,1} = 0, \end{cases} \quad (2)$$

其中, $\inf\{\cdot\}$ 表示下确界, $\Phi_i = v_i - u_i$, $\Delta_i \in (0, 1)$ 与 $l_i > 0$ 为设计参数, $t_{i,k}$ 为第 i 个智能体第 k 个触发时刻, v_i 随后给出. 事件触发控制方法表示从触发时刻 $t_{i,k}$ 到下一触发时刻 $t_{i,k+1}$ 之间, $u_i(t)$ 均保持 v_i 在 $t_{i,k}$ 时刻的控制输入, 直到下一触发时刻 $t_{i,k+1}$ 才会更新控制输入. 相对阈值策略为设计一种与控制器 u_i 幅值相关的可变阈值. 当控制信号 u_i 幅值维持较大时, 较大的阈值可以避免频繁触发; 当系统稳定, u_i 幅值维持较小时, 较小的阈值能够获得更好的控制性能.

考虑系统存在输入饱和现象, 将 v_i 表示如下^[34]:

$$v_i(w_i) = \text{sat}(w_i) = \begin{cases} \text{sign}(w_i)u_{i,N}, & |w_i| \geq u_{i,N}, \\ w_i, & |w_i| < u_{i,N}, \end{cases} \quad (3)$$

其中, $u_{i,N}$ 为控制输入的饱和阈值, 自适应控制 w_i 将在式 (30) 中给出.

为了方便处理输入饱和, 引入如下的双曲正切函数逼近式 (3),

$$p_i(w_i) = u_{i,N} \times \tanh\left(\frac{w_i}{u_{i,N}}\right) = u_{i,N} \frac{e^{w_i/u_{i,N}} - e^{-w_i/u_{i,N}}}{e^{w_i/u_{i,N}} + e^{-w_i/u_{i,N}}},$$

因此, 输入饱和函数转换为

$$\text{sat}(w_i) = p_i(w_i) + q_i(w_i) = u_{i,N} \times \tanh\left(\frac{w_i}{u_{i,N}}\right) + q_i(w_i). \quad (4)$$

模型误差 $|q_i(w_i)| = |\text{sat}(w_i) - p_i(w_i)| \leq u_{i,N}(1 - \tanh(1)) = Q_i$, 即 $q_i(w_i)$ 是上界为 Q_i 的有界函数.

本文的控制目标是: 设计事件触发控制器使跟随者的输出信号位于领导者输出信号的一个小的邻域内, 且闭环系统内所有信号是半全局一致最终有界的.

为了实现以上控制目标, 我们给出如下假设与引理.

假设1 给定跟踪轨迹信号满足 $X_0 = [y_0, \dot{y}_0, \ddot{y}_0]^T \in \Omega_d$ 光滑可导, $\Omega_d = \{X_0 : y_0^2 + \dot{y}_0^2 + \ddot{y}_0^2 \leq B_0\}$, B_0 为已知正常数.

假设2 ([35]) 未知但光滑的干扰 $d_{i,m}$ 导数有界, 即存在一个正常数 $D_{i,m}$, 使得 $|\dot{d}_{i,m}| \leq D_{i,m}$.

假设3 已知非线性函数 $g_{i,m}$ 为 L_2 范数有界, 即满足 $\|g_{i,m}\|_2 = (g_{i,m}^T g_{i,m})^{\frac{1}{2}} \leq G_{i,m}$, $G_{i,m}$ 为正常数.

引理2 (Young's 不等式^[36]) 如果 m, n, p 和 q 为正实数, 且 p, q 满足 $\frac{1}{p} + \frac{1}{q} = 1$, 那么以下不等式成立:

$$mn \leq \frac{1}{p}m^p + \frac{1}{q}n^q.$$

2.3 干扰观测器设计

为了使系统在未知干扰影响下实现良好的控制性能, 提高控制精度, 设计如下干扰观测器^[35]:

$$\begin{cases} \dot{\hat{d}}_{i,m} = \hat{z}_{i,m} + \zeta_{i,m}x_{i,m}, \\ \dot{\hat{z}}_{i,m} = -\zeta_{i,m}(x_{i,m+1} + f_{i,m} + \hat{\theta}_i^T g_{i,m} + \hat{d}_{i,m}), \end{cases} \quad (5)$$

其中, $\hat{d}_{i,m}$ 是对干扰 $d_{i,m}$ 的估计值, $\hat{\theta}_i$ 为参数 θ_i 的估计值, 定义 $\tilde{d} = d - \hat{d}$, $\tilde{\theta} = \theta - \hat{\theta}$. $\hat{z}_{i,m}$ 为干扰观测器的辅助系统状态, $\zeta_{i,m} > 0$ 为设计参数. 通过式 (5) 设计的干扰观测器, 可推导出

$$\dot{\tilde{d}}_{i,m} = \dot{d}_{i,m} - \zeta_{i,m}(\tilde{\theta}_i^T g_{i,m} + \tilde{d}_{i,m}). \quad (6)$$

第 3 节将证明干扰估计误差是一致最终有界的, 干扰观测器的有效性则在仿真中予以验证.

3 自适应动态面控制设计及稳定性分析

本节对系统 (1) 进行自适应动态面控制设计. 定义第 i 个智能体的误差面如下:

$$\tilde{x}_{i,1} = \sum_{j \in \mathcal{N}_i} a_{i,j}(y_i - y_j) + a_{i,0}(y_i - y_0), \quad (7)$$

$$\tilde{x}_{i,r} = x_{i,r} - \lambda_{i,r-1}, \quad r = 2, 3, \dots, n-1, \quad (8)$$

$$\tilde{x}_{i,n} = x_{i,n} - \lambda_{i,n-1} - \tanh(h_i), \quad (9)$$

其中, 式 (7) 为结合图论知识得出, y_0 为领导者的输出信号, h_i 为饱和补偿系统的状态, 将在式 (37) 中给出. $\lambda_{i,k}$ ($k = 1, \dots, n-1$) 为虚拟控制器 $\alpha_{i,k}$ 通过一阶低通滤波器的输出, 则一阶滤波器的输出误差为

$$\eta_{i,k} = \lambda_{i,k} - \alpha_{i,k}. \quad (10)$$

步骤 1. 构造 Lyapunov 函数为

$$V_{i,1} = \frac{1}{2}\tilde{x}_{i,1}^2 + \frac{1}{2}\tilde{d}_{i,1}^2 + \frac{1}{2}\tilde{\theta}_i^T \Gamma_i^{-1} \tilde{\theta}_i, \quad (11)$$

其中, Γ_i 是正定矩阵. 对式 (11) 两端求导, 将式 (6) 和 (7) 代入得

$$\begin{aligned} \dot{V}_{i,1} = & \tilde{x}_{i,1} \left[(b_i + a_{i,0})(\tilde{x}_{i,2} + \alpha_{i,1} + \eta_{i,1} + f_{i,1} + \theta_i^T g_{i,1} + d_{i,1}) - \sum_{j \in \mathcal{N}_i} a_{i,j}(x_{j,2} + f_{j,1} + \theta_j^T g_{j,1} + d_{j,1}) \right. \\ & \left. - a_{i,0}\dot{y}_0 \right] + \tilde{d}_{i,1}[\dot{d}_{i,1} - \zeta_{i,1}(\tilde{\theta}_i^T g_{i,1} + \tilde{d}_{i,1})] + \tilde{\theta}_i^T \Gamma_i^{-1} \dot{\tilde{\theta}}_i. \end{aligned} \quad (12)$$

由 Young's 不等式可知

$$\tilde{d}_{i,1}\dot{d}_{i,1} \leq \frac{1}{2}D_{i,1}^2 + \frac{1}{2}\tilde{d}_{i,1}^2, \quad (13)$$

$$-\zeta_{i,1}\tilde{d}_{i,1}\tilde{\theta}_i^T g_{i,1} \leq \frac{1}{2}\zeta_{i,1}\rho_{i,1}^2\tilde{\theta}_i^T \tilde{\theta}_i + \zeta_{i,1}\frac{G_{i,1}^2}{2\rho_{i,1}^2}\tilde{d}_{i,1}^2, \quad (14)$$

$$(b_i + a_{i,0})\tilde{x}_{i,1}(\tilde{d}_{i,1} + \eta_{i,1}) \leq (b_i + a_{i,0})^2\tilde{x}_{i,1}^2 + \frac{1}{2}\tilde{d}_{i,1}^2 + \frac{1}{2}\eta_{i,1}^2, \quad (15)$$

$$-\tilde{x}_{i,1} \sum_{j \in \mathcal{N}_i} a_{i,j} (\tilde{\theta}_j^T g_{j,1} + \tilde{d}_{j,1}) \leq \sum_{j \in \mathcal{N}_i} \frac{a_{i,j}}{2} (1 + g_{j,1}^T g_{j,1}) \tilde{x}_{i,1}^2 + \sum_{j \in \mathcal{N}_i} \frac{a_{i,j}}{2} (\tilde{\theta}_j^T \tilde{\theta}_j + \tilde{d}_{j,1}^2), \quad (16)$$

其中, $\rho_{i,1}$ 为正的设计参数. 设计虚拟控制器 $\alpha_{i,1}$ 为

$$\begin{aligned} \alpha_{i,1} = & -f_{i,1} - \hat{\theta}_i^T g_{i,1} - \hat{d}_{i,1} - (b_i + a_{i,0}) \tilde{x}_{i,1} + \frac{1}{b_i + a_{i,0}} \left\{ \sum_{j \in \mathcal{N}_i} a_{i,j} \left[x_{j,2} + f_{j,1} + \hat{\theta}_j^T g_{j,1} + \hat{d}_{j,1} \right. \right. \\ & \left. \left. - \frac{1}{2} (1 + g_{j,1}^T g_{j,1}) \tilde{x}_{i,1} \right] + a_{i,0} \dot{y}_0 - c_{i,1} \tilde{x}_{i,1} \right\}, \end{aligned} \quad (17)$$

其中, $c_{i,1} > 0$ 为设计参数. 将式 (13)~(17) 代入式 (12) 得

$$\begin{aligned} \dot{V}_{i,1} \leq & -c_{i,1} \tilde{x}_{i,1}^2 + (b_i + a_{i,0}) \tilde{x}_{i,1} \tilde{x}_{i,2} + \sum_{j \in \mathcal{N}_i} \frac{a_{i,j}}{2} (\tilde{\theta}_j^T \tilde{\theta}_j + \tilde{d}_{j,1}^2) - \left(\zeta_{i,1} - \zeta_{i,1} \frac{G_{i,1}^2}{2\rho_{i,1}^2} - 1 \right) \tilde{d}_{i,1}^2 + \frac{1}{2} \eta_{i,1}^2 \\ & + \frac{1}{2} D_{i,1}^2 + \frac{1}{2} \zeta_{i,1} \rho_{i,1}^2 \tilde{\theta}_i^T \tilde{\theta}_i - \tilde{\theta}_i^T [\Gamma_i^{-1} \dot{\hat{\theta}}_i - (b_i + a_{i,0}) \tilde{x}_{i,1} g_{i,1}]. \end{aligned}$$

步骤 r ($r = 2, 3, \dots, n-1$). 让虚拟控制信号 $\alpha_{i,r-1}$ 通过时间常数为 $\tau_{i,r-1}$ 的一阶低通滤波器^[37],

$$\tau_{i,r-1} \dot{\lambda}_{i,r-1} + \lambda_{i,r-1} = \alpha_{i,r-1}, \quad \lambda_{i,r-1}(0) = \alpha_{i,r-1}(0). \quad (18)$$

则由式 (10) 和 (18) 可得

$$\dot{\eta}_{i,r-1} = -\frac{\eta_{i,r-1}}{\tau_{i,r-1}} - \dot{\alpha}_{i,r-1} = -\frac{\eta_{i,r-1}}{\tau_{i,r-1}} + \psi_{i,r-1}, \quad (19)$$

其中, $\psi_{i,r-1}$ 为连续函数, 当 $r = 2$ 时, $\psi_{i,r-1}(\tilde{x}_{i,1}, \tilde{x}_{i,2}, \eta_{i,1}, \hat{d}_{i,1}, \tilde{x}_{j,2}, \eta_{j,1}, \hat{d}_{j,1}, \tilde{x}_{j,3}, \eta_{j,2}, \hat{d}_{j,2}, \hat{\theta}_i, \hat{\theta}_j, y_0, \dot{y}_0, \dot{y}_0) = \frac{\partial f_{i,1}}{\partial x_{i,1}} \dot{x}_{i,1} + \hat{\theta}_i^T g_{i,1} + \hat{\theta}_i^T \frac{\partial g_{i,1}}{\partial x_{i,1}} \dot{x}_{i,1} + \dot{\hat{d}}_{i,1} + (b_i + a_{i,0}) \dot{\tilde{x}}_{i,1} - \frac{1}{b_i + a_{i,0}} \left\{ \sum_{j \in \mathcal{N}_i} a_{i,j} \left[\dot{x}_{j,2} + \frac{\partial f_{j,1}}{\partial x_{j,1}} \dot{x}_{j,1} + \hat{\theta}_j^T g_{j,1} + \hat{\theta}_j^T \frac{\partial g_{j,1}}{\partial x_{j,1}} \dot{x}_{j,1} + \dot{\hat{d}}_{j,1} - \frac{1}{2} (1 + g_{j,1}^T g_{j,1}) \dot{\tilde{x}}_{i,1} - \tilde{x}_{i,1} g_{j,1}^T \frac{\partial g_{j,1}}{\partial x_{j,1}} \dot{x}_{j,1} \right] + a_{i,0} \dot{y}_0 - c_{i,1} \dot{\tilde{x}}_{i,1} \right\}$; 当 $r \neq 2$ 时, $\psi_{i,r-1}(\tilde{x}_{i,1}, \dots, \tilde{x}_{i,r}, \eta_{i,1}, \dots, \eta_{i,r-1}, \hat{d}_{i,1}, \dots, \hat{d}_{i,r-1}, \tilde{x}_{j,2}, \eta_{j,1}, \hat{d}_{j,1}, \tilde{x}_{j,3}, \eta_{j,2}, \hat{d}_{j,2}, \hat{\theta}_i, \hat{\theta}_j, y_0, \dot{y}_0, \dot{y}_0) = c_{i,r-1} \dot{\tilde{x}}_{i,r-1} + \varrho_{i,r-2} \dot{\tilde{x}}_{i,r-2} + \dot{\tilde{x}}_{i,r-1} + \sum_{\beta=1}^{r-1} \frac{\partial f_{i,r-1}}{\partial x_{i,\beta}} \dot{x}_{i,\beta} + \hat{\theta}_i^T g_{i,r-1} + \hat{\theta}_i^T \sum_{\beta=1}^{r-1} \frac{\partial g_{i,r-1}}{\partial x_{i,\beta}} \dot{x}_{i,\beta} + \dot{\hat{d}}_{i,r-1} - \frac{\eta_{i,r-2}}{\tau_{i,r-2}}$. 参数 $\varrho_{i,1} = b_i + a_{i,0}$; $\varrho_{i,k} = 1$ ($k \neq 1$).

构造 Lyapunov 函数为

$$V_{i,r} = V_{i,r-1} + \frac{1}{2} \tilde{x}_{i,r}^2 + \frac{1}{2} \tilde{d}_{i,r}^2 + \frac{1}{2} \eta_{i,r-1}^2. \quad (20)$$

对式 (20) 两端求导, 并将式 (6), (8) 和 (19) 代入得

$$\begin{aligned} \dot{V}_{i,r} = & \dot{V}_{i,r-1} + \tilde{x}_{i,r} (\tilde{x}_{i,r+1} + \alpha_{i,r} + \eta_{i,r} + \nu_r \tanh(h_i) + f_{i,r} + \theta_i^T g_{i,r} + d_{i,r} - \dot{\lambda}_{i,r-1}) \\ & + \eta_{i,r-1} \left(-\frac{\eta_{i,r-1}}{\tau_{i,r-1}} + \psi_{i,r-1} \right) + \tilde{d}_{i,r} [\dot{d}_{i,r} - \zeta_{i,r} (\tilde{\theta}_i^T g_{i,r} + \tilde{d}_{i,r})], \end{aligned} \quad (21)$$

其中, 参数 $\nu_{n-1} = 1$; $\nu_k = 0$ ($k \neq n-1$).

根据 Young's 不等式可得

$$\eta_{i,r-1} \psi_{i,r-1} \leq \frac{1}{2} \psi_{i,r-1}^2 + \frac{1}{2} \eta_{i,r-1}^2, \quad (22)$$

$$-\zeta_{i,r} \tilde{d}_{i,r} \tilde{\theta}_i^T g_{i,r} \leq \frac{1}{2} \zeta_{i,r} \rho_{i,r}^2 \tilde{\theta}_i^T \tilde{\theta}_i + \zeta_{i,r} \frac{G_{i,r}^2}{2\rho_{i,r}^2} \tilde{d}_{i,r}^2, \quad (23)$$

$$\tilde{x}_{i,r}(\tilde{d}_{i,r} + \eta_{i,r}) + \tilde{d}_{i,r}\dot{\tilde{x}}_{i,r} \leq \tilde{x}_{i,r}^2 + \tilde{d}_{i,r}^2 + \frac{1}{2}\eta_{i,r}^2 + \frac{1}{2}D_{i,r}^2, \quad (24)$$

其中, $\rho_{i,r}$ 为正的设计参数. 令设计参数 $c_{i,r} > 0$, 虚拟控制器 $\alpha_{i,r}$ 设计为

$$\alpha_{i,r} = -c_{i,r}\tilde{x}_{i,r} - \varrho_{i,r-1}\tilde{x}_{i,r-1} - \tilde{x}_{i,r} - f_{i,r} - \hat{\theta}_i^T g_{i,r} - \tilde{d}_{i,r} + \dot{\lambda}_{i,r-1} - \nu_r \tanh\left(\frac{\tilde{x}_{i,n-1}}{\epsilon_{i,1}}\right). \quad (25)$$

将式 (22)~(25) 代入式 (21) 可得

$$\begin{aligned} \dot{V}_{i,r} \leq & -\sum_{\beta=1}^r c_{i,\beta}\tilde{x}_{i,\beta}^2 + \tilde{x}_{i,r} \left[\tilde{x}_{i,r+1} + \nu_r \tanh(h_i) - \nu_r \tanh\left(\frac{\tilde{x}_{i,n-1}}{\epsilon_{i,1}}\right) \right] + \sum_{j \in \mathcal{N}_i} \frac{a_{i,j}}{2} (\tilde{\theta}_j^T \tilde{\theta}_j + \tilde{d}_{j,1}^2) \\ & - \sum_{\beta=1}^r \left(\zeta_{i,\beta} - \zeta_{i,\beta} \frac{G_{i,\beta}^2}{2\rho_{i,\beta}^2} - 1 \right) \tilde{d}_{i,\beta}^2 + \frac{1}{2} \sum_{\gamma=1}^{r-1} \psi_{i,\gamma}^2 - \sum_{\gamma=1}^{r-1} \left(\frac{1}{\tau_{i,\gamma}} - 1 \right) \eta_{i,\gamma}^2 + \frac{1}{2} \sum_{\beta=1}^r \zeta_{i,\beta} \rho_{i,\beta}^2 \tilde{\theta}_i^T \tilde{\theta}_i \\ & - \tilde{\theta}_i^T \left(\Gamma_i^{-1} \dot{\tilde{\theta}}_i - \sum_{\beta=1}^r \varrho_{i,\beta} \tilde{x}_{i,\beta} g_{i,\beta} \right) + \frac{1}{2} \sum_{\beta=1}^r D_{i,\beta}^2 + \frac{1}{2} \eta_{i,r}^2. \end{aligned}$$

步骤 n . 让虚拟控制信号 $\alpha_{i,n-1}$ 通过时间常数为 $\tau_{i,n-1}$ 的一阶低通滤波器, 即

$$\tau_{i,n-1} \dot{\lambda}_{i,n-1} + \lambda_{i,n-1} = \alpha_{i,n-1}, \quad \lambda_{i,n-1}(0) = \alpha_{i,n-1}(0). \quad (26)$$

则由式 (10) 和 (26) 可得

$$\dot{\eta}_{i,n-1} = -\frac{\eta_{i,n-1}}{\tau_{i,n-1}} - \dot{\alpha}_{i,n-1} = -\frac{\eta_{i,n-1}}{\tau_{i,n-1}} + \psi_{i,n-1}, \quad (27)$$

其中, $\psi_{i,n-1}$ 为连续函数, $\psi_{i,n-1}(\tilde{x}_{i,1}, \dots, \tilde{x}_{i,n}, \eta_{i,1}, \dots, \eta_{i,n-1}, \hat{d}_{i,1}, \dots, \hat{d}_{i,n-1}, \tilde{x}_{j,2}, \eta_{j,1}, \hat{d}_{j,1}, \tilde{x}_{j,3}, \eta_{j,2}, \hat{d}_{j,2}, \hat{\theta}_i, \hat{\theta}_j, y_0, \dot{y}_0, \ddot{y}_0) = c_{i,n-1}\dot{\tilde{x}}_{i,n-1} + \dot{\tilde{x}}_{i,n-2} + \dot{\tilde{x}}_{i,n-1} + \sum_{\beta=1}^{n-1} \frac{\partial f_{i,n-1}}{\partial x_{i,\beta}} \dot{x}_{i,\beta} + \dot{\hat{\theta}}_i^T g_{i,n-1} + \hat{\theta}_i^T \sum_{\beta=1}^{n-1} \frac{\partial g_{i,n-1}}{\partial x_{i,\beta}} \dot{x}_{i,\beta} + \dot{\tilde{d}}_{i,n-1} - \frac{\dot{\eta}_{i,n-2}}{\tau_{i,n-2}} - \frac{\dot{\tilde{x}}_{i,n-1}}{\cosh^2(\tilde{x}_{i,n-1}/\epsilon_{i,1})}$.

构造 Lyapunov 函数为

$$V_{i,n} = V_{i,n-1} + \frac{1}{2}\tilde{x}_{i,n}^2 + \frac{1}{2}\tilde{d}_{i,n}^2 + \frac{1}{2}\eta_{i,n-1}^2. \quad (28)$$

对式 (28) 两端求导, 并将式 (6), (9) 和 (27) 代入得

$$\begin{aligned} \dot{V}_{i,n} = & \dot{V}_{i,n-1} + \tilde{x}_{i,n} \left(u_i + f_{i,n} + \theta_i^T g_{i,n} + d_{i,n} - \dot{\lambda}_{i,n-1} - \frac{1}{\cosh^2(h_i)} \dot{h}_i \right) + \tilde{d}_{i,n} [\dot{d}_{i,n} - \zeta_{i,n}(\tilde{\theta}_i^T g_{i,n} + \tilde{d}_{i,n})] \\ & + \eta_{i,n-1} \left(-\frac{\eta_{i,n-1}}{\tau_{i,n-1}} + \psi_{i,n-1} \right). \end{aligned} \quad (29)$$

设计自适应控制 w_i 为

$$w_i = -(1 + \Delta_i) \left[\alpha_{i,n} \tanh\left(\frac{\tilde{x}_{i,n}\alpha_{i,n}}{\epsilon_{i,2}}\right) + \bar{l}_i \tanh\left(\frac{\tilde{x}_{i,n}\bar{l}_i}{\epsilon_{i,2}}\right) \right], \quad (30)$$

其中, 虚拟控制器 $\alpha_{i,n}$ 随后给出, $\bar{l}_i > \frac{l_i}{1-\Delta_i}$, ϵ_i 为正的设计参数.

根据式 (2) 定义的触发机制可知, 当 $u_i \geq 0$ 时, $u_i = \frac{v_i - \delta_{i,1}l_i}{1 + \delta_{i,1}\Delta_i}$; 当 $u_i < 0$ 时, $u_i = \frac{v_i - \delta_{i,1}l_i}{1 - \delta_{i,1}\Delta_i}$. 时变函数 $\delta_{i,1}(t)$ 满足 $|\delta_{i,1}(t)| \leq 1$, 令 $\delta_{i,2}(t) = \text{sign}(u_i)\delta_{i,1}(t)$. 由于 $\tilde{x}_{i,n}w_i \leq 0$ 成立, 进而 $\tilde{x}_{i,n}v_i \leq 0$ 成立, 则

$$\tilde{x}_{i,n}u_i = \frac{\tilde{x}_{i,n}(v_i - \delta_{i,1}l_i)}{1 + \delta_{i,2}\Delta_i} \leq \frac{\tilde{x}_{i,n}v_i}{1 + \Delta_i} + \frac{|\tilde{x}_{i,n}l_i|}{1 - \Delta_i} \leq \frac{\tilde{x}_{i,n}v_i}{1 + \Delta_i} + |\tilde{x}_{i,n}\bar{l}_i|. \quad (31)$$

根据 Young's 不等式可得

$$\psi_{i,n-1}\eta_{i,n-1} \leq \frac{1}{2}\psi_{i,n-1}^2 + \frac{1}{2}\eta_{i,n-1}^2, \quad (32)$$

$$\tilde{d}_{i,n}\dot{d}_{i,n} + \tilde{x}_{i,n}\tilde{d}_{i,n} \leq \tilde{d}_{i,n}^2 + \frac{1}{2}\tilde{x}_{i,n}^2 + \frac{1}{2}D_{i,n}^2, \quad (33)$$

$$-\zeta_{i,n}\tilde{d}_{i,n}\tilde{\theta}_i^T g_{i,n} \leq \frac{1}{2}\zeta_{i,n}\rho_{i,n}^2\tilde{\theta}_i^T\tilde{\theta}_i + \zeta_{i,n}\frac{G_{i,n}^2}{2\rho_{i,n}^2}\tilde{d}_{i,n}^2, \quad (34)$$

$$\frac{\tilde{x}_{i,n}q_i(w_i)}{1+\Delta_i} \leq \frac{\tilde{x}_{i,n}^2}{2(1+\Delta_i)} + \frac{Q_i^2}{2(1+\Delta_i)}, \quad (35)$$

其中, $\rho_{i,n}$ 为正的设计参数. 将式 (4), (31)~(35) 代入式 (29) 可得

$$\begin{aligned} \dot{V}_{i,n} \leq & \dot{V}_{i,n-1} + \tilde{x}_{i,n} \left[\frac{p_i(w_i)}{1+\Delta_i} + f_{i,n} + \hat{\theta}_i^T g_{i,n} + \hat{d}_{i,n} - \dot{\lambda}_{i,n-1} - \frac{1}{\cosh^2(h_i)}\dot{h}_i + \frac{1}{2}\tilde{x}_{i,n} + \frac{1}{2(1+\Delta_i)}\tilde{x}_{i,n} \right] \\ & - \left(\frac{1}{\tau_{i,n-1}} - \frac{1}{2} \right) \eta_{i,n-1}^2 - \left(\zeta_{i,n} - \zeta_{i,n}\frac{G_{i,n}^2}{2\rho_{i,n}^2} - 1 \right) \tilde{d}_{i,n}^2 + \frac{1}{2}\zeta_{i,n}\rho_{i,n}^2\tilde{\theta}_i^T\tilde{\theta}_i + \tilde{x}_{i,n}\tilde{\theta}_i^T g_{i,n} \\ & + \frac{1}{2}\psi_{i,n-1}^2 + \frac{1}{2}D_{i,n}^2 + \frac{Q_i^2}{2(1+\Delta_i)} + |\tilde{x}_{i,n}\bar{l}_i|. \end{aligned} \quad (36)$$

为了补偿输入饱和对系统稳定性的影响, 设计饱和补偿系统为

$$\dot{h}_i = \cosh^2(h_i) \left(-h_i - \frac{w_i}{1+\Delta_i} + \frac{p_i(w_i)}{1+\Delta_i} \right). \quad (37)$$

由 $\tanh(\cdot)$ 函数性质有

$$0 \leq |z| - z \tanh\left(\frac{z}{\epsilon}\right) \leq 0.2785\epsilon,$$

其中, $\epsilon > 0, z \in \mathbb{R}$. 则以下不等式成立

$$\tilde{x}_{i,n-1} \tanh(h_i) - \tilde{x}_{n-1} \tanh\left(\frac{\tilde{x}_{i,n-1}}{\epsilon_{i,1}}\right) \leq |\tilde{x}_{i,n-1}| - \tilde{x}_{n-1} \tanh\left(\frac{\tilde{x}_{i,n-1}}{\epsilon_{i,1}}\right) \leq 0.2785\epsilon_{i,1}, \quad (38)$$

$$\begin{aligned} \frac{\tilde{x}_{i,n}w_i}{1+\Delta_i} &= \tilde{x}_{i,n}\alpha_{i,n} - \tilde{x}_{i,n}\alpha_{i,n} \tanh\left(\frac{\tilde{x}_{i,n}\alpha_{i,n}}{\epsilon_{i,2}}\right) + |\tilde{x}_{i,n}\bar{l}_i| - \tilde{x}_{i,n}\bar{l}_i \tanh\left(\frac{\tilde{x}_{i,n}\bar{l}_i}{\epsilon_{i,2}}\right) - \tilde{x}_{i,n}\alpha_{i,n} - |\tilde{x}_{i,n}\bar{l}_i| \\ &\leq 0.557\epsilon_{i,2} - \tilde{x}_{i,n}\alpha_{i,n} - |\tilde{x}_{i,n}\bar{l}_i|. \end{aligned} \quad (39)$$

令虚拟控制器 $\alpha_{i,n}$ 及自适应律 $\dot{\hat{\theta}}_i$ 分别为

$$\alpha_{i,n} = c_{i,n}\tilde{x}_{i,n} + \tilde{x}_{i,n-1} + \frac{1}{2}\tilde{x}_{i,n} + \frac{1}{2(1+\Delta_i)}\tilde{x}_{i,n} + f_{i,n} + \hat{\theta}_i^T g_{i,n} + \hat{d}_{i,n} - \dot{\lambda}_{i,n-1} + h_i, \quad (40)$$

$$\dot{\hat{\theta}}_i = \Gamma_i \left(\sum_{\beta=1}^n \varrho_{i,\beta}\tilde{x}_{i,\beta}g_{i,\beta} - \sigma_i\hat{\theta}_i \right). \quad (41)$$

将式 (37)~(41) 的结果代入式 (36) 可得

$$\dot{V}_{i,n} \leq - \sum_{\beta=1}^n c_{i,\beta}\tilde{x}_{i,\beta}^2 - \sum_{\beta=1}^n \left(\zeta_{i,\beta} - \zeta_{i,\beta}\frac{G_{i,\beta}^2}{2\rho_{i,\beta}^2} - 1 \right) \tilde{d}_{i,\beta}^2 - \sum_{\gamma=1}^{n-1} \left(\frac{1}{\tau_{i,\gamma}} - 1 \right) \eta_{i,\gamma}^2 + \frac{1}{2} \sum_{\beta=1}^n \zeta_{i,\beta}\rho_{i,\beta}^2\tilde{\theta}_i^T\tilde{\theta}_i + \sigma_i\hat{\theta}_i^T\hat{\theta}_i$$

$$+ \sum_{j \in \mathcal{N}_i} \frac{a_{i,j}}{2} (\tilde{\theta}_j^T \tilde{\theta}_j + \tilde{d}_{j,1}^2) + \frac{1}{2} \sum_{\gamma=1}^{n-1} \psi_{i,\gamma}^2 + \frac{1}{2} \sum_{\beta=1}^n D_{i,\beta}^2 + \frac{Q_i^2}{2(1+\Delta_i)} + 0.2785(\epsilon_{i,1} + 2\epsilon_{i,2}).$$

多智能体系统的 Lyapunov 函数为

$$V = \sum_{i=1}^N V_{i,n} = \sum_{i=1}^N \sum_{\beta=1}^n \frac{1}{2} \tilde{x}_{i,\beta}^2 + \sum_{i=1}^N \sum_{\beta=1}^n \frac{1}{2} \tilde{d}_{i,\beta}^2 + \sum_{i=1}^N \sum_{\beta=1}^{n-1} \frac{1}{2} \eta_{i,\beta}^2 + \sum_{i=1}^N \frac{1}{2} \tilde{\theta}_i^T \Gamma_i^{-1} \tilde{\theta}_i. \quad (42)$$

由于 $\sum_{i=1}^N \sigma_i \tilde{\theta}_i^T \hat{\theta}_i \leq -\sum_{i=1}^N \frac{\sigma_i}{2} \tilde{\theta}_i^T \tilde{\theta}_i + \sum_{i=1}^N \frac{\sigma_i}{2} \theta_i^T \theta_i$, $\sum_{i=1}^N \sum_{j \in \mathcal{N}_i} \frac{a_{i,j}}{2} (\tilde{\theta}_j^T \tilde{\theta}_j + \tilde{d}_{j,1}^2) = \sum_{i=1}^N \sum_{j=1}^N \frac{a_{j,i}}{2} \cdot (\tilde{\theta}_i^T \tilde{\theta}_i + \tilde{d}_{i,1}^2)$ 成立. 那么, 对式 (42) 两端求导可得

$$\begin{aligned} \dot{V} \leq & - \sum_{i=1}^N \sum_{\beta=1}^n c_{i,\beta} \tilde{x}_{i,\beta}^2 - \sum_{i=1}^N \left(\zeta_{i,1} - \zeta_{i,1} \frac{G_{i,1}^2}{2\rho_{i,1}^2} - 1 - \sum_{j=1}^N \frac{a_{j,i}}{2} \right) \tilde{d}_{i,1}^2 - \sum_{i=1}^N \sum_{\beta=2}^n \left(\zeta_{i,\beta} - \zeta_{i,\beta} \frac{G_{i,\beta}^2}{2\rho_{i,\beta}^2} - 1 \right) \tilde{d}_{i,\beta}^2 \\ & - \sum_{i=1}^N \sum_{\gamma=1}^{n-1} \left(\frac{1}{\tau_{i,\gamma}} - 1 \right) \eta_{i,\gamma}^2 - \sum_{i=1}^N \frac{\sigma_i - \sum_{\varpi=1}^n \zeta_{i,\varpi} \rho_{i,\varpi}^2 - \sum_{j=1}^N a_{j,i}}{2\lambda_{\max}(\Gamma_i^{-1})} \tilde{\theta}_i^T \Gamma_i^{-1} \tilde{\theta}_i + \sum_{i=1}^N \frac{\sigma_i}{2} \theta_i^T \theta_i + \frac{1}{2} \sum_{i=1}^N \sum_{\gamma=1}^{n-1} \psi_{i,\gamma}^2 \\ & + \frac{1}{2} \sum_{i=1}^N \sum_{\beta=1}^n D_{i,\beta}^2 + \sum_{i=1}^N \frac{Q_i^2}{2(1+\Delta_i)} + \sum_{i=1}^N 0.2785(\epsilon_{i,1} + 2\epsilon_{i,2}), \end{aligned} \quad (43)$$

其中, $\lambda_{\max}(\cdot)$ 表示矩阵的最大特征值. 对任意 $B_0 > 0$, $P > 0$, $\Omega_{i,k} := \{ \sum_{\gamma=1}^i [\tilde{\theta}_\gamma^T \Gamma_\gamma^{-1} \tilde{\theta}_\gamma + \sum_{\beta=1}^k (\frac{1}{2} \tilde{x}_{\gamma,\beta}^2 + \frac{1}{2} \tilde{d}_{\gamma,\beta}^2) + \sum_{\beta=1}^{k-1} \frac{1}{2} \eta_{\gamma,\beta}^2] + \sum_{\varpi \in \mathcal{N}_i} \tilde{x}_{\varpi,k+1}^2 \leq 2P \}$ 为 $\mathbb{R}^{\dim(\Omega_{i,k})}$ 内的有界紧集, $i = 1, 2, \dots, N$, $k = 1, 2, \dots, n$, $\dim(\Omega_{i,k})$ 表示 $\Omega_{i,k}$ 的维度. 那么 $|\psi_{i,k}|$ 在有界紧集 $\Omega_d \times \Omega_{i,k}$ 上有最大值 $A_{i,k}$ [38, 39].

选择参数 $\zeta_{i,1}$, $\zeta_{i,\beta}$, $\rho_{i,1}$, $\rho_{i,\beta}$, $\tau_{i,\gamma}$ 和 σ_i 使 $\zeta_{i,1} - \zeta_{i,1} \frac{G_{i,1}^2}{2\rho_{i,1}^2} - 1 - \sum_{j=1}^N \frac{a_{j,i}}{2} > 0$, $\zeta_{i,\beta} - \zeta_{i,\beta} \frac{G_{i,\beta}^2}{2\rho_{i,\beta}^2} - 1 > 0$, $\frac{1}{\tau_{i,\gamma}} - 1 > 0$, $\sigma_i - \sum_{\varpi=1}^n \zeta_{i,\varpi} \rho_{i,\varpi}^2 - \sum_{j=1}^N a_{j,i} > 0$ 成立. 令 $\kappa = \frac{1}{2} \sum_{i=1}^N \sum_{\gamma=1}^{n-1} A_{i,\gamma}^2 + \frac{1}{2} \sum_{i=1}^N \sum_{\beta=1}^n D_{i,\beta}^2 + \sum_{i=1}^N \frac{Q_i^2}{2(1+\Delta_i)} + \sum_{i=1}^N 0.2785(\epsilon_{i,1} + 2\epsilon_{i,2}) + \frac{1}{2} \sum_{i=1}^N \sigma_i \theta_i^T \theta_i$, $\mu = \min\{2c_{i,1}, 2c_{i,\beta}, 2\zeta_{i,1} - \zeta_{i,1} \frac{G_{i,1}^2}{\rho_{i,1}^2} - 2 - \sum_{j=1}^N a_{j,i}, 2\zeta_{i,\beta} - \zeta_{i,\beta} \frac{G_{i,\beta}^2}{\rho_{i,\beta}^2} - 2, \frac{2}{\tau_{i,\gamma}} - 2, \frac{\sigma_i - \sum_{\varpi=1}^n \zeta_{i,\varpi} \rho_{i,\varpi}^2 - \sum_{j=1}^N a_{j,i}}{\lambda_{\max}(\Gamma_i^{-1})}\}$, $i = 1, 2, \dots, N$, $\gamma = 1, 2, \dots, n-1$, $\beta = 2, 3, \dots, n$. 则式 (43) 可重写为

$$\dot{V} \leq -\mu V + \kappa. \quad (44)$$

由式 (44) 可知, 当 $\mu > \kappa/P$ 时, 如果 $V = P$, 则 $\dot{V} < 0$. 因此, $V(t) \leq P$ 是一个不变集, 意味着如果 $V(0) \leq P$ 成立, 那么 $V(t) \leq P, \forall t > 0$ [40, 41]. 根据式 (44) 可证, 闭环系统内所有信号是半全局一致最终有界的, 且可推得如下不等式:

$$\frac{1}{2} \tilde{x}_{i,1}^2 \leq V \leq \frac{\kappa}{\mu} + \left(V(0) - \frac{\kappa}{\mu} \right) e^{-\mu t}. \quad (45)$$

令 $\tilde{x}_1 = [\tilde{x}_{1,1}, \dots, \tilde{x}_{N,1}]^T$, $\bar{y} = [y_1, \dots, y_N]^T$, $\bar{y}_0 = [y_0, \dots, y_0]^T$ (y_0 个数为 N), $\tilde{y} = \bar{y} - \bar{y}_0$. 由式 (7) 可得 [42]

$$\begin{aligned} \tilde{x}_{i,1} &= \sum_{j \in \mathcal{N}_i} a_{i,j} [(y_i - y_0) - (y_j - y_0)] + a_{i,0} (y_i - y_0), \\ \tilde{x}_1 &= \mathcal{B} \tilde{y} - \mathcal{A} \tilde{y} + \mathcal{C} \tilde{y} = (\mathcal{L} + \mathcal{C}) \tilde{y}, \\ \|\tilde{y}\| &= \|(\mathcal{L} + \mathcal{C})^{-1} \tilde{x}_1\| \leq \frac{\|\tilde{x}_1\|}{\varsigma(\mathcal{L} + \mathcal{C})}, \end{aligned} \quad (46)$$

图 1 通信拓扑图

Figure 1 Communication topology

其中, $\varsigma(\cdot)$ 表示矩阵的最小奇异值. 则由式 (45) 和 (46) 有

$$\lim_{t \rightarrow \infty} \|\tilde{y}\| \leq \frac{1}{\varsigma(\mathcal{L} + \mathcal{C})} \sqrt{\frac{2\kappa}{\mu}},$$

则通过选择合适的参数改变 μ 和 κ 使一致跟踪误差收敛在原点附近一个小的邻域内.

Zeno 行为是一种有限时间内发生无数次事件触发的现象. 如果事件被触发无数次就无法节约资源, 系统也难以稳定. 现在证明提出的控制方法存在最小触发时间间隔从而避免 Zeno 行为的发生.

$$\frac{d}{dt}|\Phi_i| = \text{sign}(\Phi_i) \frac{d(\Phi_i)}{dt} \leq |\dot{w}_i|,$$

由于 w_i 可微, 且 $\dot{w}_i$ 的自变量均为有界闭环信号, 那么必存在一个正常数 ξ_i 使得 $|\dot{w}_i| \leq \xi_i$. 因为 $\Phi_i(t_{i,k}) = 0$, $\lim_{t \rightarrow t_{i,k+1}} |\Phi_i(t)| > l_i$, 即触发时间间隔一定大于 l_i/ξ_i , 因此可以避免 Zeno 行为的发生. 通过以上分析, 本文的主要结果总结为如下定理.

定理1 对于多智能体系统 (1), 在满足假设 1~3, 且对于任意 $P > 0$, 初始条件满足 $V(0) \leq P$ 时, 通过干扰观测器 (5)、饱和补偿系统 (37)、自适应律 (41) 和经式 (2), (4), (30) 和 (40) 设计的事件触发控制器, 选择适当的设计参数 $c_{i,1}, \dots, c_{i,n}, \tau_{i,1}, \dots, \tau_{i,n-1}, \Gamma_i, \sigma_i, \epsilon_{i,1}, \epsilon_{i,2}, \Delta_i, l_i, \bar{l}_i$ 可使闭环系统稳定, 所有信号是半全局一致最终有界的, 并且一致跟踪误差收敛在原点附近一个小的邻域内.

4 仿真算例

本节将由仿真 1 (数值仿真) 和仿真 2 (实例仿真) 两部分来验证本文控制算法的有效性. 本文仿真的多智能体系统由 1 个领导者和 4 个跟随者组成, 依次编号为 0, 1, 2, 3, 4. 领导者的输出信号为 $y_0 = \frac{1}{2} \sin(t)$, 通信拓扑图如图 1 所示. 假设所有边的权重都为 1, 那么由图可知, $\mathcal{C} = \text{diag}\{1, 0, 1, 0\}$, 跟随者的权重邻接矩阵 $\mathcal{A}$ 与 Laplace 矩阵 $\mathcal{L}$ 分别为

$$\mathcal{A} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix}, \quad \mathcal{L} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 0 & 0 & 1 & -1 \\ -1 & 0 & 0 & 1 \end{bmatrix}.$$

仿真 1. 首先应用数值仿真验证本文提出控制算法的有效性, 每个跟随者的系统动力学方程描述如下:

$$\dot{x}_{i,1} = x_{i,2} + x_{i,1} \sin(x_{i,1}) + \theta_i x_{i,1} \cos(x_{i,1}) + d_{i,1},$$

图 2 (网络版彩图) 仿真 1 跟随者的跟踪效果

Figure 2 (Color online) Tracking effect of followers in simulation 1. (a) Outputs of followers and the leader; (b) tracking errors

图 3 (网络版彩图) 仿真 1 干扰的估计误差

Figure 3 (Color online) Estimation errors of disturbances $d_{i,1}$ (a) and $d_{i,2}$ (b) in simulation 1

$$\begin{aligned}\dot{x}_{i,2} &= u_i + \frac{1}{2}x_{i,2} \cos(x_{i,1}^2) + \frac{1}{4}\theta_i x_{i,2} \sin(x_{i,2}^2) + d_{i,2}, \\ y_i &= x_{i,1}, \quad i = 1, 2, 3, 4.\end{aligned}$$

未知干扰假定为 $d_{i,1} = \cos(t)$, $d_{i,2} = \frac{1}{2} - \frac{1}{2} \cos(2t)$. 部分初始条件设定为 $x_1(0) = [-0.05, 0.1]^T$, $x_2(0) = [-0.05, -0.11]^T$, $x_3(0) = [-0.05, 0.2]^T$, $x_4(0) = [-0.05, 0.1]^T$. $\hat{z}_1(0) = [3.75, -6.5]^T$, $\hat{z}_2(0) = [4, 7.7]^T$, $\hat{z}_3(0) = [25, -16]^T$, $\hat{z}_4(0) = [4.5, -8.5]^T$. 选取设计参数为 $u_{i,N} = 20$, $\theta_i = 0.02$, $c_{i,1} = 80$, $c_{i,2} = 40$, $\tau_{i,1} = 0.01$, $\epsilon_{i,1} = \epsilon_{i,2} = 2.6$, $\Delta_i = 0.08$, $l_i = 0.01$, $\Gamma_i = 0.03$, $\sigma_i = 100$, $\bar{l}_i = 1$. $[\zeta_{1,1}, \zeta_{2,1}, \zeta_{3,1}, \zeta_{4,1}] = [55, 60, 65, 70]$; $[\zeta_{1,2}, \zeta_{2,2}, \zeta_{3,2}, \zeta_{4,2}] = [65, 70, 80, 85]$.

数值仿真结果如图 2~5 所示. 图 2 为跟随者的跟踪效果图, 由图 2(b) 的跟踪误差示意图可知, 经过约 1 s 的调整时间后, 一致跟踪误差收敛在区间 $[-0.005, 0.005]$ 内. 图 3 为未知干扰 $d_{i,1}$, $d_{i,2}$ 的估计误差示意图, 由图可知, 选取不同的参数 $\zeta_{i,1}$, $\zeta_{i,2}$ 可以调整误差的范围, 验证了干扰观测器的有效性. 图 4 为事件触发控制器信号图, 可以看到 u_i 的幅值维持在 $u_{i,N}$ 内, 局部放大图显示控制信号仅在满足触发条件的时刻更新, 表明本文提出方法能够有效降低对通信资源的要求. 并且可以看出应用相对阈值策略时, 触发阈值随 u_i 的变化而变化, 当 u_i 幅值接近 0 时, 触发阈值随之减小. 图 5 为控制输入的事件触发时间间隔示意图, 横轴表示发生触发的时刻, 纵轴表示该次触发持续时间, 每一框图上方显示总触发次数, 表明没有发生 Zeno 行为. 因此, 数值仿真验证了所提出的控制方法的有效性.

仿真 2. 将本文控制方法进一步应用到文献 [43] 的机械臂系统. 受到未知干扰影响的系统动力学方程如下:

$$\begin{aligned}\dot{x}_{i,1} &= x_{i,2} + d_{i,1}, \\ \dot{x}_{i,2} &= \frac{1}{J_i} u_i + \theta_i^T [x_{i,2}; \sin(x_{i,1})] + d_{i,2}, \\ y_i &= x_{i,1}, \quad i = 1, 2, 3, 4,\end{aligned}$$

其中, $x_{i,1}$, $x_{i,2}$ 分别为连杆角速度和角加速度, $M_i = 1 \text{ kg}$ 为连杆总质量, $g = 10 \text{ m/s}^2$ 为重力加

图 4 (网络版彩图) 仿真 1 事件触发控制器

Figure 4 (Color online) Event-triggered controller in simulation 1

图 5 仿真 1 事件触发的时间间隔

Figure 5 Time interval of triggering events in simulation 1. (a) u_1 ; (b) u_2 ; (c) u_3 ; (d) u_4

图 6 (网络版彩图) 仿真 2 跟随者的跟踪效果

Figure 6 (Color online) Tracking effect of followers in simulation 2. (a) Outputs of followers and the leader; (b) tracking errors

图 7 (网络版彩图) 仿真 2 干扰的估计误差

Figure 7 (Color online) Estimation errors of disturbances $\tilde{d}_{i,1}$ (a) and $\tilde{d}_{i,2}$ (b) in simulation 2

速度, $L_i = 1$ m 为从关节轴到连杆质心的距离, $D_i = 2$ N·m·s/rad 为总阻尼系数, $J_i = 1$ kg·m² 为连杆总转动惯量, 未知系统参数 $\theta_i = [-\frac{D_i}{J_i}, -\frac{M_i g L_i}{J_i}]^T = [-2, -10]^T$, 未知干扰假定为 $d_{i,1} = \cos(t)$, $d_{i,2} = 4 \sin(t+1.4)$. 部分初始条件为 $x_1(0) = [-0.05, 0.1]^T$, $x_2(0) = [-0.1, -0.11]^T$, $x_3(0) = [-0.09, 0.1]^T$, $x_4(0) = [-0.1, -0.1]^T$. $\hat{z}_1(0) = [4.3, 4]^T$, $\hat{z}_2(0) = [8.5, 4]^T$, $\hat{z}_3(0) = [8.2, 3.89]^T$, $\hat{z}_4(0) = [9.5, 4]^T$. 选取设计参数为 $u_{i,N} = 50$, $c_{i,1} = 80$, $c_{i,2} = 60$, $\tau_{i,1} = 0.018$, $\epsilon_{i,1} = 2.6$, $\epsilon_{i,2} = 4$, $\Delta_i = 0.06$, $l_i = 0.08$, $\sigma_i = 100$, $\bar{l}_i = 1$, $\Gamma_i = [0.01, 0; 0, 0.01]$, $[\zeta_{1,1}, \zeta_{2,1}, \zeta_{3,1}, \zeta_{4,1}] = [70, 75, 80, 85]$, $[\zeta_{1,2}, \zeta_{2,2}, \zeta_{3,2}, \zeta_{4,2}] = [0.81, 0.81, 0.81, 0.81]$.

实例仿真结果如图 6~9 所示. 由仿真结果可知, 干扰观测器能够有效地估计未知干扰, u_i 的幅值维持在 $u_{i,N}$ 内, 系统稳定时, 跟踪误差的上界为 0.006. 因此, 本文提出的控制算法应用于实际机械臂

图 8 (网络版彩图) 仿真 2 事件触发控制器

Figure 8 (Color online) Event-triggered controller in simulation 2

图 9 仿真 2 事件触发的时间间隔

Figure 9 Time interval of triggering events in simulation 2. (a) u_1 ; (b) u_2 ; (c) u_3 ; (d) u_4

系统同样有效.

5 结论

本文针对一类含有未知系统参数和输入饱和的多智能体系统,研究了其在有向拓扑图下的领导跟随一致性问题.首先,设计了一种干扰观测器,能够有效估计未知干扰.其次,构建饱和补偿系统解决了系统输入饱和时的控制问题,提高了系统的控制性能.然后,利用动态面控制技术避免了反步法固有的“计算爆炸”问题.最后,基于事件触发机制解决了通信资源有限的问题.两个仿真算例验证了本文方法是有效的.未来的工作考虑将本文的研究推广到更具普遍性的多智能体系统.

参考文献

- Jadbabaie A, Lin J, Morse A S. Coordination of groups of mobile autonomous agents using nearest neighbor rules. *IEEE Trans Autom Control*, 2003, 48: 988–1001
- Ma L F, Wang Z D, Han Q L, et al. Consensus control of stochastic multi-agent systems: a survey. *Sci China Inf Sci*, 2017, 60: 120201
- Li T S, Zhao R, Chen C L P, et al. Finite-time formation control of under-actuated ships using nonlinear sliding mode control. *IEEE Trans Cybern*, 2018, 48: 3243–3253
- Qian W, Gao Y S, Yang Y. Global consensus of multiagent systems with internal delays and communication delays. *IEEE Trans Syst Man Cybern Syst*, 2019, 49: 1961–1970
- Qian W, Wang L, Chen M Z Q. Local consensus of nonlinear multiagent systems with varying delay coupling. *IEEE Trans Syst Man Cybern Syst*, 2018, 48: 2462–2469
- Lu Z H, Zhang L, Wang L. Controllability analysis of multi-agent systems with switching topology over finite fields. *Sci China Inf Sci*, 2019, 62: 012201
- Xu J J, Xu L, Xie L H, et al. Decentralized control for linear systems with multiple input channels. *Sci China Inf Sci*, 2019, 62: 052202
- Gao Z Q. On the foundation of active disturbance rejection control. *Control Theory Appl*, 2013, 30: 1498–1510 [高志强. 自抗扰控制思想探究. *控制理论与应用*, 2013, 30: 1498–1510]
- Guo L, Cao S Y. Anti-disturbance control theory for systems with multiple disturbances: a survey. *ISA Trans*, 2014, 53: 846–849

- 10 Chen M, Tao G. Adaptive fault-tolerant control of uncertain nonlinear large-scale systems with unknown dead zone. *IEEE Trans Cybern*, 2016, 46: 1851–1862
- 11 Chen W H, Ballance D J, Gawthrop P J, et al. A nonlinear disturbance observer for robotic manipulators. *IEEE Trans Ind Electron*, 2000, 47: 932–938
- 12 Sun W, Su S F, Xia J W, et al. Adaptive tracking control of wheeled inverted pendulums with periodic disturbances. *IEEE Trans Cybern*, 2018. doi: 10.1109/TCYB.2018.2884707
- 13 Chen W H, Yang J, Guo L, et al. Disturbance-observer-based control and related methods — an overview. *IEEE Trans Ind Electron*, 2016, 63: 1083–1095
- 14 Liu X Y, Ho D W C, Cao J D, et al. Discontinuous observers design for finite-time consensus of multiagent systems with external disturbances. *IEEE Trans Neur Netw Learn Syst*, 2017, 28: 2826–2830
- 15 Ding Z T. Consensus disturbance rejection with disturbance observers. *IEEE Trans Ind Electron*, 2015, 62: 5829–5837
- 16 Li G P, Wang X Y, Li S H. Distributed composite output consensus protocols of higher-order multi-agent systems subject to mismatched disturbances. *IET Control Theory Appl*, 2017, 11: 1162–1172
- 17 Hu X, Wei X J, Zhang H F, et al. Robust adaptive tracking control for a class of mechanical systems with unknown disturbances under actuator saturation. *Int J Robust Nonlinear Control*, 2019, 29: 1893–1908
- 18 Chen M, Shao S Y, Jiang B. Adaptive neural control of uncertain nonlinear systems using disturbance observer. *IEEE Trans Cybern*, 2017, 47: 3110–3123
- 19 Lin Z L. Control design in the presence of actuator saturation: from individual systems to multi-agent systems. *Sci China Inf Sci*, 2019, 62: 026201
- 20 Bai W W, Zhou Q, Li T S, et al. Adaptive reinforcement learning neural network control for uncertain nonlinear system with input saturation. *IEEE Trans Cybern*, 2019. doi: 10.1109/TCYB.2019.2921057
- 21 Qin J H, Fu W M, Zheng W X, et al. On the bipartite consensus for generic linear multiagent systems with input saturation. *IEEE Trans Cybern*, 2017, 47: 1948–1958
- 22 Su H S, Chen M Z Q. Multi-agent containment control with input saturation on switching topologies. *IET Control Theor Appl*, 2015, 9: 399–409
- 23 Wang X L, Su H S, Chen M Z Q, et al. Observer-based robust coordinated control of multiagent systems with input saturation. *IEEE Trans Neur Netw Learn Syst*, 2018, 29: 1933–1946
- 24 Dimarogonas D V, Frazzoli E, Johansson K H. Distributed event-triggered control for multi-agent systems. *IEEE Trans Autom Control*, 2012, 57: 1291–1297
- 25 Cao L, Li H Y, Zhou Q. Adaptive intelligent control for nonlinear strict-feedback systems with virtual control coefficients and uncertain disturbances based on event-triggered mechanism. *IEEE Trans Cybern*, 2018, 48: 3390–3402
- 26 Xing L T, Wen C Y, Liu Z T, et al. Event-triggered adaptive control for a class of uncertain nonlinear systems. *IEEE Trans Autom Control*, 2017, 62: 2071–2076
- 27 Wang C L, Guo L, Qiao J Z. Event-triggered adaptive fault-tolerant control for nonlinear systems fusing static and dynamic information. *J Franklin Inst*, 2019, 356: 248–267
- 28 Ma H, Li H Y, Liang H J, et al. Adaptive fuzzy event-triggered control for stochastic nonlinear systems with full state constraints and actuator faults. *IEEE Trans Fuzzy Syst*, 2019. doi: 10.1109/TFUZZ.2019.2896843
- 29 Hu Q L, Shi Y X, Wang C L. Event-based formation coordinated control for multiple spacecraft under communication constraints. *IEEE Trans Syst Man Cybern Syst*, 2019. doi: 10.1109/TSMC.2019.2919027
- 30 Wang A J, Liao X F, He H B. Event-triggered differentially private average consensus for multi-agent network. *IEEE/CAA J Autom Sin*, 2019, 6: 75–83
- 31 Zou W C, Xiang Z R. Event-triggered containment control of second-order nonlinear multi-agent systems. *J Franklin Inst*, 2018. doi: 10.1016/j.jfranklin.2018.05.060
- 32 Zhang Y H, Sun J, Liang H J, et al. Event-triggered adaptive tracking control for multiagent systems with unknown disturbances. *IEEE Trans Cybern*, 2018. doi: 10.1109/TCYB.2018.2869084
- 33 Wang W, Wen C Y, Huang J S. Distributed adaptive asymptotically consensus tracking control of nonlinear multi-agent systems with unknown parameters and uncertain disturbances. *Automatica*, 2017, 77: 133–142
- 34 Zhou Q, Wang L J, Wu C W, et al. Adaptive fuzzy control for nonstrict-feedback systems with input saturation and output constraint. *IEEE Trans Syst Man Cybern Syst*, 2017, 47: 1–12
- 35 Du J L, Hu X, Krstić M, et al. Robust dynamic positioning of ships with disturbances under input saturation.

- Automatica, 2016, 73: 207–214
- 36 Zhou Q, Zhao S Y, Li H Y, et al. Adaptive neural network tracking control for robotic manipulators with dead zone. IEEE Trans Neur Netw Learn Syst, 2018. doi: 10.1109/TNNLS.2018.286937
 - 37 Li Y M, Li K W, Tong S C. Finite-time adaptive fuzzy output feedback dynamic surface control for MIMO nonstrict feedback systems. IEEE Trans Fuzzy Syst, 2019, 27: 96–110
 - 38 Yoo S J. Connectivity-preserving consensus tracking of uncertain nonlinear strict-feedback multiagent systems: an error transformation approach. IEEE Trans Neur Netw Learn Syst, 2018, 29: 4542–4548
 - 39 Li Y M, Tong S C. Adaptive fuzzy control with prescribed performance for block-triangular-structured nonlinear systems. IEEE Trans Fuzzy Syst, 2018, 26: 1153–1163
 - 40 Swaroop D, Hedrick J K, Yip P P, et al. Dynamic surface control for a class of nonlinear systems. IEEE Trans Autom Control, 2000, 45: 1893–1899
 - 41 Wang C L, Guo L. Adaptive cooperative tracking control for a class of nonlinear time-varying multi-agent systems. J Franklin Inst, 2017, 354: 6766–6782
 - 42 Zhang H W, Lewis F L. Adaptive cooperative tracking control of higher-order nonlinear systems with unknown dynamics. Automatica, 2012, 48: 1432–1439
 - 43 Xing L T, Wen C Y, Liu Z T, et al. Adaptive compensation for actuator failures with event-triggered input. Automatica, 2017, 85: 129–136

Disturbance-observer-based event-triggered control for multi-agent systems with input saturation

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Abstract With the aim to solve the leader-following consensus problem for a class of strict-feedback multi-agent systems with unknown disturbances and input saturation under a directed topology, this paper proposes a distributed event-triggered control approach based on disturbance observer. First, this approach designs a disturbance observer to estimate unknown disturbances, and then combining the dynamic surface control with the adaptive backstepping approach, the control input is obtained. Second, a relative threshold event-triggered control strategy is adopted to solve the problem of limited communication resources. Using the saturation compensation system, a mismatch between the controller signal and the actuator signal is compensated. Besides, using the Lyapunov stability theory, it can be proved that all signals of the closed-loop system are semiglobally uniformly ultimately bounded. Finally, simulation is conducted to evaluate the performance of the proposed approach, and the simulation results prove the effectiveness of the proposed control approach.

Keywords multi-agent systems, disturbance observer, input saturation, event-triggered control, dynamic surface control

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